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IMPROVED AND EXTENDED MIXED-RADIX DECIMATION IN FREQUENCY FAST MDCT ALGORITHM

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Abstract. Recently, a mixed-radix decimation in frequency (DIF) fast MDCT algorithm only for the mixed-radix decompositions or composite lengths $N = 3^m \times 2$, $m > 0$, has been proposed in [4]. An improved mixed-radix DIF fast MDCT algorithm both in terms of the regularity and computational complexity is described. Based on observed simple algebraic identities in the original proposed algorithm [4], new formulas are derived resulting in a very regular computational structure. Consequently, the number of arithmetic operations is reduced significantly. Moreover, the improved algorithm is extended to all composite lengths $N = 3^m \times 2^p$, $m, p > 0$. The improved algorithm defines new sparse matrix factorizations of the MDCT matrix for the composite lengths $N = 3^m \times 2^p$, $m, p > 0$, and finally it provides new implementations of the forward/backward MDCT in MPEG-1/2 layer III (MP3) audio coding standard.

Keywords: Modified discrete cosine transform, mixed-radix fast algorithm, MP3 audio coding

Mathematics Subject Classification 2000: 68W40, 68Q25

1 INTRODUCTION

The modified discrete cosine transform (MDCT) [1] has become the fundamental processing block in the current international audio coding standards and commercial digital audio compression algorithms for high-quality compression/decompression of digital audio signals in consumer electronics [2, 3]. Since the forward and backward MDCT computation are the most time-consuming operations in audio codecs, an efficient implementation of the MDCT processor has become the key technology to realize real-time low-cost and low-power audio decoders in (portable) audio players and digital multimedia systems. In particular, with the popularity of MPEG-1/2 layer III audio coding standard known as MP3, where the size of an audio data block is $N = 12$ (the short block) or $N = 36$ (the long block), much research has been devoted to develop an efficient implementation of the MDCT in MP3. A comprehensive list of references covering various fast MDCT algorithms and hardware implementations developed and adopted in the last decade for the efficient MDCT implementation in MP3 can be found in [6, 7]. Among the recently proposed fast algorithms which can be adopted for the efficient MDCT implementation in MP3 are two mixed-radix fast MDCT algorithms: the first one obtained by the "decimation in frequency" (DIF) decomposition method only for the composite lengths $N = 3^m \times 2$, $m > 0$ [4], and the second one obtained by the "decimation in time" (DIT) decomposition method only for the composite lengths $N = 3^m \times 4$, $m > 0$ [5].

In this paper, an improved mixed-radix DIF fast MDCT algorithm both in terms of the regularity and computational complexity is described. Based on observed simple algebraic identities in the original proposed algorithm [4], new formulas are derived resulting in a very regular computational structure. Consequently, the number of arithmetic operations is reduced significantly. Moreover, the improved algorithm is extended to all mixed-radix decompositions or composite lengths $N = 3^m \times 2^p$, $m, p > 0$. This fact allows to combine mixed-radix DIF and DIT fast MDCT algorithms with a recursive radix-2 fast MDCT algorithm [5] to further widespread existing choices of sequence lengths as well as to construct a variety of the MDCT implementations with a different fast computational structure and a different computational complexity. The improved algorithm defines new sparse matrix factorizations of the MDCT matrix for the composite lengths $N = 3^m \times 2^p$, $m, p > 0$. Finally, it provides new implementations of the forward/backward MDCT in MP3 audio coding standard.

It is important to note that the analysis of computational structure and associated arithmetic complexity of the mixed-radix fast MDCT algorithm published in [4] is incomplete and is not properly/fully investigated. Further, some errors occurring in the published paper [4] are corrected. Essentially, compared to [4], the improved mixed-radix DIF fast MDCT algorithm with the complete computational analysis for all composite lengths $N = 3^m \times 2^p$, $m, p > 0$ is presented here.

2 DEFINITIONS AND MIXED-RADIX DIF FAST MDCT ALGORITHM

The forward and backward MDCT block transforms are, respectively, defined as [1]

$$\begin{aligned} c_k &= \sqrt{\frac{4}{N}} \sum_{n=0}^{N-1} x_n \cos\left[\frac{\pi}{2N}(2n+1+\frac{N}{2})(2k+1)\right], \quad k = 0,1,\ldots,\frac{N}{2}-1, \\ \hat{x}_n &= \sqrt{\frac{4}{N}} \sum_{k=0}^{\frac{N}{2}-1} c_k \cos\left[\frac{\pi}{2N}(2n+1+\frac{N}{2})(2k+1)\right], \quad n = 0,1,\ldots,N-1. \end{aligned} \tag{1}$$

The input data sequence $\{x_n\}$ in (1) is assumed to be windowed by a windowing function before its transformation. $\{\hat{x}_n\}$ in (1) represents the time-domain aliased data sequence recovered by the backward MDCT block transform which does not correspond to the original data sequence $\{x_n\}$.

Recently, the mixed-radix DIF fast MDCT algorithm for composite lengths $N = 3^m \times 2$, $m > 0$, has been proposed in [4]. Importantly, the strictly defined radix-3 MDCT algorithm cannot be constructed since from the MDCT definition it follows that 3^m is not divisible by 2. Complete formulas of the original mixed-radix DIF fast MDCT algorithm are expressed in the more convenient form as

$$c_{3k+1} = \sum_{n=0}^{\frac{N}{3}-1} (x_{\frac{N}{3}-1-n} - x_{\frac{2N}{3}-1-n} + x_{N-1-n}) \cos\phi_{n,k}, \quad k = 0,1,\ldots,\frac{N}{6}-1, \tag{2}$$

and introducing the following expressions $a_k = c_{3k} + c_{3k+2}$ and $b_k = c_{3k} - c_{3k+2}$, for $k = 0,1,\ldots,\frac{N}{6}-1$, where

$$\begin{aligned} a_k &= -\sum_{n=0}^{\frac{N}{3}-1} \Big[(x_{\frac{N}{3}-1-n} + x_{\frac{2N}{3}-1-n})\sqrt{3}\cos\frac{\pi(2n+1)}{N} \qquad (3) \\ &\quad + (x_{\frac{N}{3}-1-n} - x_{\frac{2N}{3}-1-n} - 2x_{N-1-n})\sin\frac{\pi(2n+1)}{N} \Big] \cos\phi_{n,k}, \\ b_k &= (-1)^k \sum_{n=0}^{\frac{N}{3}-1} \Big[(2x_n + x_{\frac{N}{3}+n} - x_{\frac{2N}{3}+n})\cos\frac{\pi(2n+1)}{N} \qquad (4) \\ &\quad + (x_{\frac{N}{3}+n} + x_{\frac{2N}{3}+n})\sqrt{3}\sin\frac{\pi(2n+1)}{N} \Big] \cos\phi_{n,k}, \\ \phi_{n,k} &= \frac{\pi}{2(N/3)}(2n+1+\frac{N}{6})(2k+1), \qquad k = 0,1,\ldots,\frac{N}{6}-1, \qquad (5) \end{aligned}$$

and taking into account Equation (2) the complete set of MDCT coefficients is obtained as

$$c_{3k} = \frac{1}{2}(a_k + b_k), \qquad c_{3k+2} = \frac{1}{2}(a_k - b_k), \qquad k = 0,1,\ldots,\frac{N}{6}-1. \tag{6}$$

Thus, the N-point MDCT is obtained from the computation of three $\frac{N}{3}$-point MDCTs. The factors $\frac{1}{2}$ from (6) can be simply absorbed into Equation (4). For composite lengths $N = 3^m \times 2$, $m > 0$, the arithmetic complexity of the original mixed-radix DIF fast algorithm [4] without any optimization (M_N is the number of multiplications and A_N is the number of additions) is given by

$$M_N = 3 \times M_{\frac{N}{3}} + \frac{4N}{3}, \qquad A_N = 3 \times A_{\frac{N}{3}} + \frac{10N}{3},$$

where for $N = 3 \times 2 = 6$, $M_2 = A_2 = 0$. Note that $\frac{N}{3}$ multiplications by 2 are implicitly taken as shift operations in [4].

The main motivation to improve and extend the proposed mixed-radix DIF fast MDCT algorithm is based on the following essential facts implied from the original paper [4]:

- Mixed-radix DIF fast MDCT algorithm in [4] is defined only for composite lengths $N = 3^m \times 2$, $m > 0$. Additionally, a derived mixed-radix DIF fast algorithm for the backward MDCT computation with composite lengths $N = 3^m \times 4$, $m > 0$, is almost redundant.
- There exists an error in Equation (10) of [4], where the "–" sign is forgotten before the sum.
- The analysis of computational structure and associated arithmetic complexity of algorithm [4] for the lengths $N = 3^m \times 2$, $m > 0$, is incomplete and is not properly/fully investigated.
- A signal flow graph only for the 6-point forward MDCT computation has been presented in [4] and, moreover, it is not correct (only the first two butterfly stages are correct).

In particular, it can be seen from Equations (2) and (4) that all algebraic expressions between round brackets combined with cosine/sine twiddle factors are quite different, and, consequently, the regularity of a computational structure of the algorithm is not clear. Possibly, this is probably the reason why the authors in [4] did not further investigate the computational structure of their algorithm in detail.

3 IMPROVED AND EXTENDED MIXED-RADIX DIF FAST MDCT ALGORITHM

The key result for the derivation of improved mixed-radix DIF fast MDCT algorithm is an observation that each algebraic expression between round brackets under the first sum of (4) corresponding to $\{a_k\}$ can be derived by a proper combination of two algebraic expressions between round brackets under the second sum of (4) corresponding to $\{b_k\}$. Specifically, denoting

$$u_n = 2x_n + x_{\frac{N}{3}+n} - x_{\frac{2N}{3}+n}, \qquad v_n = x_{\frac{N}{3}+n} + x_{\frac{2N}{3}+n}, \tag{7}$$

the following simple algebraic identities hold:

$$\begin{aligned} x_{\frac{N}{3}-1-n} + x_{\frac{2N}{3}-1-n} &= \frac{1}{2}(u_{\frac{N}{3}-1-n} + v_{\frac{N}{3}-1-n}), \\ x_{\frac{N}{3}-1-n} - x_{\frac{2N}{3}-1-n} - 2x_{N-1-n} &= \frac{1}{2}(u_{\frac{N}{3}-1-n} - 3v_{\frac{N}{3}-1-n}), n = 0, 1 \ldots, \frac{N}{3} - 1. \end{aligned} \tag{8}$$

At first, substituting the algebraic identities (8) into the first sum of (4) subsequently followed by substituting $n' = \frac{N}{3} - 1 - n$ for n, after some algebraic manipulations we get a new very regular form of the improved mixed-radix DIF fast MDCT algorithm defined as

$$c_{3k+1} = \sum_{n=0}^{\frac{N}{3}-1} \left[x_{\frac{N}{3}-1-n} - (x_{\frac{2N}{3}-1-n} - x_{N-1-n})\right] \cos\phi_{n,k}, \quad k = 0, 1, \ldots, \frac{N}{6} - 1, \tag{9}$$

with

$$\begin{aligned} a_k &= -\sum_{n=0}^{\frac{N}{3}-1} \left[u_n \sin\frac{\pi(2n+1)}{N} - v_n\sqrt{3}\cos\frac{\pi(2n+1)}{N}\right] \cos\phi_{\frac{N}{3}-1-n,k}, \\ b_k &= (-1)^k \sum_{n=0}^{\frac{N}{3}-1} \left[u_n \cos\frac{\pi(2n+1)}{N} + v_n\sqrt{3}\sin\frac{\pi(2n+1)}{N}\right] \cos\phi_{n,k}, \\ & k = 0, 1, \ldots, \frac{N}{6} - 1. \end{aligned} \tag{10}$$

The complete set of MDCT coefficients is obtained from (6). The factors $\frac{1}{2}$ in (6) can be simply absorbed into Equation (10). Due to the algorithm regularity the expression between round brackets in (2) is rewritten into the more convenient equivalent form in Equation (9). Comparing Equations (4) and (10) one can see that the algebraic expressions (u_n and v_n) combined with the sine/cosine twiddle factors in Equation (10) are the same. This fact enables us to investigate the computational structure of the algorithm in detail for all the composite lengths $N = 3^m \times 2^p$, $m, p > 0$. The cosine transform kernel $\cos\phi_{\frac{N}{3}-1-n,k}$ in the first sum of (10) is recognized as the MDCT transform kernel in the reverse order, whereby $\cos\phi_{\frac{N}{3}-1-n,k} = (-1)^{k+1}\sin\phi_{n,k}$. For a given N the computation of higher-order MDCTs is obtained from recursively reused three lower-order $\frac{N}{3}$-point MDCTs. The backward MDCT computation can be simply realized by reversing a fast computational structure for the forward MDCT and performing the inverse operations.

4 ANALYSIS OF THE COMPUTATIONAL COMPLEXITY

In general, the computational complexity of improved mixed-radix DIF fast MDCT algorithm for the composite lengths $N = 3^m \times 2^p$, $m, p > 0$, is given mainly by the computational complexity of three $\frac{N}{3}$-point MDCTs associated with the value of p, and the computational complexity of (10). The best existing 2^p-point ($p > 1$) MDCT algorithm requires $\frac{N}{4}(p+1)$ multiplications and $\frac{N}{4}(3p-1)$ additions [6]. The evaluation of algebraic expressions in (7) requires $\frac{4N}{3}$ additions, whereby $\frac{N}{3}$ multiplications by 2 are counted as additions (note that the multiplications by 2 can be efficiently implemented as shift operations). The evaluation of algebraic expressions between the square brackets in (9) requires only $\frac{N}{3}$ additions because the sub-expressions $(x_{\frac{2N}{3}-1-n} - x_{N-1-n})$ are pre-computed in (7). To compute $\{a_k\}$ and $\{b_k\}$ in (10) we need $\frac{4N}{3}$ multiplications and $\frac{2N}{3}$ additions. Finally, to obtain the complete set of MDCT coefficients from (6) we need $\frac{N}{3}$ additions. Then, the total arithmetic complexity of the improved mixed-radix DIF fast MDCT algorithm for the composite lengths $N = 3^m \times 2^p$, $m > 0$, $p > 2$, is given by

$$M_N = 3 \times M_{\frac{N}{3}} + \frac{4N}{3}, \qquad A_N = 3 \times A_{\frac{N}{3}} + \frac{8N}{3},$$

where for $N = 3 \times 2^3$, we get $M_8 = 8$ and $A_8 = 16$. Compared to the total arithmetic complexity of the original algorithm [4], $\frac{2N}{3}$ additions are saved in the improved algorithm. For the composite lengths $N = 3^m \times 2^p$, where $p = 1$ and $p = 2$, the number of arithmetic operations for some special angles can be further reduced separately as follows.

4.1 Mixed-Radix Decompositions $N = 3^m \times 2$, $m > 0$

Let the factors $\frac{1}{2}$ in (6) be absorbed into Equation (10). The evaluation of (10) for one n needs 4 multiplications and 2 additions. For $n = \frac{1}{2}(\frac{N}{6} - 1)$ the angle is $\frac{\pi}{6}$, then $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $\sin\frac{\pi}{6} = \frac{1}{2}$, and the expressions between square brackets in Equation (10) are:

$$\begin{aligned} u_n \frac{1}{2}\sin\frac{\pi}{6} - v_n \frac{\sqrt{3}}{2}\cos\frac{\pi}{6} &= \frac{1}{4}(u_n - 3v_n), \\ u_n \frac{1}{2}\cos\frac{\pi}{6} + v_n \frac{\sqrt{3}}{2}\sin\frac{\pi}{6} &= \frac{\sqrt{3}}{4}(u_n + v_n), \end{aligned}$$

requiring 1 multiplication, 4 additions and 1 shift, where multiplication by 3 is realized by 2 additions. On the other hand, for $n = \frac{N}{6} + \frac{1}{2}(\frac{N}{6} - 1)$ the angle is $\frac{\pi}{2}$ (this case is not considered at all in [4]), and therefore the expressions between square brackets in Equation (10) are:

$$\begin{aligned} u_n \frac{1}{2}\sin\frac{\pi}{2} - v_n \frac{\sqrt{3}}{2}\cos\frac{\pi}{2} &= \frac{1}{2}u_n, \\ u_n \frac{1}{2}\cos\frac{\pi}{2} + v_n \frac{\sqrt{3}}{2}\sin\frac{\pi}{2} &= \frac{\sqrt{3}}{2}v_n, \end{aligned}$$

requiring 1 multiplication and 1 shift, so totally saving 6 multiplications. Then, the total arithmetic complexity for the composite lengths $N = 3^m \times 2, m > 0$, is given by

$$M_N = 3 \times M_{\frac{N}{3}} + \frac{4N}{3} - 6, \qquad A_N = 3 \times A_{\frac{N}{3}} + \frac{8N}{3},$$

where for $N = 3 \times 2 = 6$, $M_6 = 2$, $A_6 = 16$ plus 2 shifts (compare with the arithmetic complexity $M_6 = 5$, $A_6 = 15$ in [4]). The correct signal flow graph for the 6-point forward MDCT computation is shown in Figure 1. Exploiting the fact that the transform kernel of 2-point forward MDCT is $\cos \frac{\pi}{2}$ and $\cos \pi$ (and hence $M_2 = A_2 = 0$), the 6-point forward MDCT computation can be optimized in terms of the arithmetic complexity. All redundant computations are indicated by the thicker lines in the signal flow graph in Figure 1. Removing these redundant computations results in a new 6-point forward MDCT module with the arithmetic complexity $M_6 = 1$, $A_6 = 11$ (note that two multiplications by 2 are counted as 2 additions) plus 1 shift. Since the 6-point forward MDCT is recursively reused for the computation of the higher-order forward MDCTs, the total arithmetic complexity is reduced significantly. Comparison of the arithmetic complexity for the original and improved DIF fast MDCT algorithms for some selected composite lengths $N = 3^m \times 2, m > 0$ is summarized in Table 1.

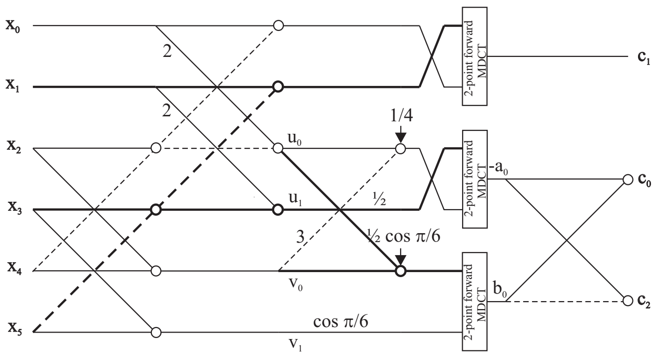

Fig. 1. Signal flow graph for the 6-point forward MDCT computation. Redundant computations are indicated by the thicker lines.

Mixed-radix decomposition of N	Original algorithm [4]		Improved algorithm		
	Mults	Adds	Mults	Adds	Shifts
$3 \times 2 = 6$	5	15	1	11	1
$3^2 \times 2 = 3 \times 6 = 18$	36	100	21	81	3
$3^3 \times 2 = 3 \times 18 = 54$	177	475	129	387	9
$3^4 \times 2 = 3 \times 54 = 162$	744	1 960	597	1 593	27

Table 1. Comparison of the arithmetic complexity for the original and improved DIF fast MDCT algorithms for some selected composite lengths $N = 3^m \times 2$, $m > 0$

Note. The number of additions for composite lengths $N = 3^m \times 2$, $m > 0$, can be further reduced using the optimized efficient 6/3-point forward/backward MDCT modules [5] generated directly from the MDCT matrix-vector representation having the arithmetic complexity 1 multiplication, 6/4 additions and 1 shift.

4.2 Mixed-Radix Decompositions $N = 3^m \times 4$, $m > 0$

Again, let the factors $\frac{1}{2}$ in (6) be absorbed into Equation (10). For $n = \frac{1}{2}(\frac{N}{4} - 1)$ the angle is $\frac{\pi}{4}$, and the expressions between square brackets in Equation (10) are:

$$\begin{aligned} u_n \frac{1}{2} \sin \frac{\pi}{4} - v_n \frac{\sqrt{3}}{2} \cos \frac{\pi}{4} &= \frac{\sqrt{2}}{4}(u_n - \sqrt{3} v_n), \\ u_n \frac{1}{2} \cos \frac{\pi}{4} + v_n \frac{\sqrt{3}}{2} \sin \frac{\pi}{4} &= \frac{\sqrt{2}}{4}(u_n + \sqrt{3} v_n), \end{aligned}$$

requiring 3 multiplications and 2 additions, so totally saving 1 multiplication. Then, the total arithmetic complexity for the composite lengths $N = 3^m \times 4, m > 0$ is given by

$$M_N = 3 \times M_{\frac{N}{3}} + \frac{4N}{3} - 1, \qquad A_N = 3 \times A_{\frac{N}{3}} + \frac{8N}{3},$$

where for $N = 3 \times 4 = 12$, $M_4 = 3$, $A_4 = 5$. The short and long block sizes in MP3 are just the composite lengths $N = 3^m \times 4$ for $m = 1$ and $m = 2$, respectively. The regular generalized signal flow graph for 12-point forward MDCT computation is shown in Figure 2. The total arithmetic complexity of the 12-point forward MDCT is 24 multiplications and 47 additions, whereby 4 multiplications by 2 are counted as 4 additions. Since the 12-point forward MDCT is recursively reused for the 36-point forward MDCT, the total arithmetic complexity of the 36-point forward MDCT computation is 119 multiplications and 237 additions whereby 24 multiplications by 2 are counted as 24 additions. Comparison of the arithmetic complexity for the original and improved DIF fast MDCT algorithms for the composite lengths $N = 3^m \times 4$, $m = 1, 2$ is summarized in Table 2.

Mixed-radix decomposition of N	Original algorithm [4]		Improved algorithm	
	Mults	Adds	Mults	Adds
$3 \times 4 = 12$	28	52	24	47
$3^2 \times 4 = 3 \times 12 = 36$	132	276	119	237

Table 2. Comparison of the arithmetic complexity for the original and improved DIF fast MDCT algorithms for the composite lengths $N = 3^m \times 4$, $m = 1, 2$.

5 NEW MDCT IMPLEMENTATIONS IN MP3

The improved mixed-radix DIF fast algorithm provides new implementations of the forward/backward MDCT in MP3 audio coding standard. The main multiplicative

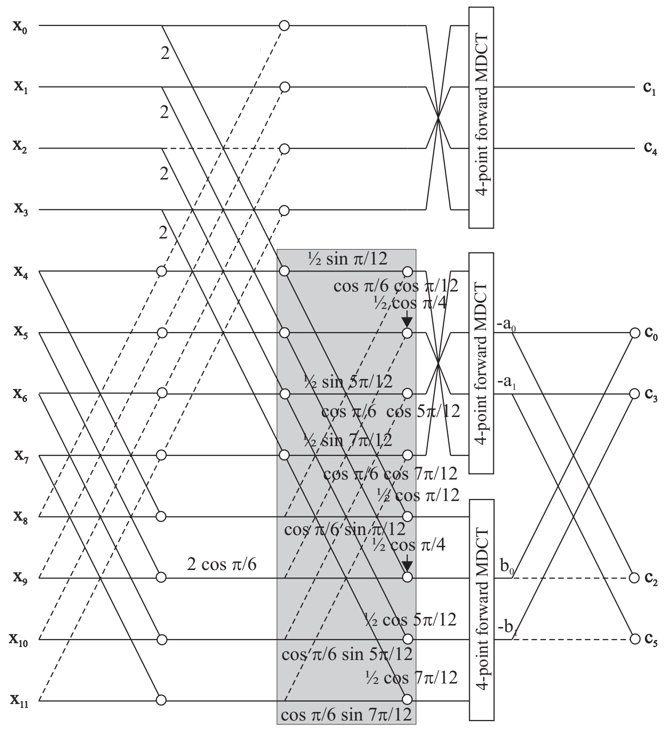

Fig. 2. Generalized signal flow graph for 12-point forward MDCT computation

complexity is concentrated in Equation (10) where the implementation for $N = 12$ (requiring 15 multiplications and 8 additions) is highlighted by shaded box in Figure 2. It was observed that the multiplicative complexity as well as the number of unique angles can be further reduced by an optimization procedure presented below which is valid only for $N = 12$.

Consider algebraic expressions between the square brackets under two sums of Equation (10) for $N = 12$ and $n = 0, 1, 2, 3$. Let the factors $\frac{1}{2}$ in (6) are absorbed into Equation (10). Since $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $\sin\frac{\pi}{6} = \frac{1}{2}$, the expressions can be written as

$$\begin{aligned} &u_n \sin\frac{\pi}{6} \sin\frac{\pi(2n+1)}{12} - v_n \cos\frac{\pi}{6} \cos\frac{\pi(2n+1)}{12}, \\ &u_n \sin\frac{\pi}{6} \cos\frac{\pi(2n+1)}{12} + v_n \cos\frac{\pi}{6} \sin\frac{\pi(2n+1)}{12}, \quad n = 0, 1, 2, 3. \end{aligned}$$

Using trigonometric identities and exploiting the fact that $N = 12$ is divisible by 6, after some algebraic manipulations, we get

$$(u_n - v_n)\tfrac{1}{2} \cos\left[\tfrac{\pi(2-(2n+1))}{12}\right] - (u_n + v_n)\tfrac{1}{2} \cos\left[\tfrac{\pi(2+(2n+1))}{12}\right],$$

$$(u_n - v_n)\frac{1}{2}\sin\left[\frac{\pi(2-(2n+1))}{12}\right] + (u_n + v_n)\frac{1}{2}\sin\left[\frac{\pi(2+(2n+1))}{12}\right],$$
$$n = 0, 1, 2, 3.$$

Subsequently, substituting $n = 0, 1, 2, 3$ into above expressions, respectively, we have

$$(u_0 - v_0)\frac{1}{2}\cos\frac{\pi}{12} - (u_0 + v_0)\frac{1}{2}\cos\frac{\pi}{4},$$
$$(u_0 - v_0)\frac{1}{2}\sin\frac{\pi}{12} + (u_0 + v_0)\frac{1}{2}\cos\frac{\pi}{4},$$

$$(u_1 - \sqrt{3}v_1)\frac{1}{2}\cos\frac{\pi}{4},$$
$$(u_1 + \sqrt{3}v_1)\frac{1}{2}\cos\frac{\pi}{4},$$

$$(u_2 - v_2)\frac{1}{2}\cos\frac{\pi}{4} + (u_2 + v_2)\frac{1}{2}\sin\frac{\pi}{12},$$
$$-(u_2 - v_2)\frac{1}{2}\cos\frac{\pi}{4} + (u_2 + v_2)\frac{1}{2}\cos\frac{\pi}{12},$$

$$(u_3 - v_3)\frac{1}{2}\sin\frac{\pi}{12} + (u_3 + v_3)\frac{1}{2}\cos\frac{\pi}{4},$$
$$-(u_3 - v_3)\frac{1}{2}\cos\frac{\pi}{12} + (u_3 + v_3)\frac{1}{2}\cos\frac{\pi}{4},$$

requiring 12 multiplications and 14 additions, so saving 3 multiplications at the cost of 6 more additions. Thus, the optimized computation of 12-point forward MDCT requires totally 21 multiplications and 53 additions, whereby 4 multiplications by 2 are counted as 4 additions. Since the 12-point forward MDCT is recursively reused as a basic computational module for the 36-point forward MDCT computation, the total arithmetic complexity of the 36-point forward MDCT computation is 110 multiplications and 255 additions, whereby 24 multiplications by 2 are counted as 24 additions. Comparison of MDCT implementations in MP3 (without optimization and optimized) based on the improved mixed-radix DIF fast algorithm and efficient MDCT implementations based on the associated mixed-radix DIT fast algorithm [5] in terms of the arithmetic complexity is summarized in Table 3.

MDCT algorithm	$N = 12$		$N = 36$	
	Mults	Adds	Mults	Adds
Improved mixed-radix DIF without optimization	24	47	119	237
Optimized improved mixed-radix DIF	21	53	110	255
Mixed-radix DIT [5]	11	27	55	141

Table 3. Comparison of MDCT implementations in MP3 (without optimization and optimized) based on the improved mixed-radix DIF fast algorithm and efficient MDCT implementations based on the associated mixed-radix DIT fast algorithm [5] in terms of the arithmetic complexity.

The backward MDCT computation requires exactly $\frac{N}{2}$ additions less than that of the forward MDCT. The correctness of improved mixed-radix DIF fast MDCT algorithm optimized for the composite lengths $N = 3 \times 4 = 12$ and $N = 3 \times 12 = 36$ has been verified by the computer program in C.

6 DISCUSSION AND CONCLUSIONS

The improved mixed-radix DIF fast MDCT algorithm both in terms of the regularity and computational complexity has been described. Based on the observed simple algebraic identities in the original proposed algorithm [4], the new formulas have been derived resulting in very regular computational structure. This fact enabled us to investigate the computational structure of the improved algorithm in detail and has lead to the following important and new results/conclusions:

- Although the improved mixed-radix DIF fast MDCT algorithm is not so efficient in terms of the computational complexity compared to the best fast MDCT algorithms [6, 7], it shares all properties of the associated mixed-radix DIT fast MDCT algorithm [5] with respect to the design criteria such as data access scheme, modularity, regularity, in-place implementation and basic computational module sharing.
- The improved algorithm has been extended to all the composite lengths $N = 3^m \times 2^p$, $m, p > 0$. This fact allows to combine mixed-radix DIF and DIT fast MDCT algorithms with the recursive radix-2 MDCT algorithm [5] to further widespread existing choices of sequence lengths as well as to construct a variety of the MDCT implementations with a different fast computational structure and a different computational complexity.
- The number of arithmetic operations has been reduced significantly compared to [4], in particular for the composite lengths $N = 3^m \times 2$ and $N = 3^m \times 4$, $m > 0$.
- The improved algorithm provides new implementations of the forward/backward MDCT in MP3 audio coding standard, where the 12-point MDCT is recursively reusable for the 36-point MDCT computation so reducing hardware resources in a potential hardware implementation.
- The improved algorithm defines new sparse matrix factorizations of the MDCT matrix for composite lengths $N = 3^m \times 2^p$, $m, p > 0$.

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