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VARIABLE PRECISION ROUGH SET MODEL FOR INCOMPLETE INFORMATION SYSTEMS AND ITS β-REDUCTS

Zengtai Gong, Zhanhong Shi, Hongxia Yao

College of Mathematics and Statistics
Northwest Normal University
730070 Lanzhou, P.R. China
e-mail: zt-gong@163.com, gongzt@nwnu.edu.cn

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Abstract. As the original rough set model is quite sensitive to noisy data, Ziarko proposed the variable precision rough set (VPRS) model to deal with noisy data and uncertain information. This model allowed for some degree of uncertainty and misclassification in the mining process. In this paper, the variable precision rough set model for an incomplete information system is proposed by combining the VPRS model and incomplete information system, and the β-lower and β-upper approximations are defined. Considering that classical VPRS model lacks a feasible method to determine the precision parameter β when calculating the β-reducts, we present an approach to determine the parameter β. Then, by calculating discernibility matrix and discernibility functions based on β-lower approximation, the β-reducts and the generalized decision rules are obtained. Finally, a concrete example is given to explain the validity and practicability of β-reducts which is proposed in this paper.

Keywords: Variable precision rough sets, incomplete information systems, approximation space, tolerance relation, β-reducts

Mathematics Subject Classification 2010: 93C41, 68U35

1 INTRODUCTION

Rough set theory (RST) was proposed by Pawlak in 1982 [11]. It is an excellent mathematical tool for dealing with vague, uncertain, imprecise and incomplete information. With more than twenty years development, RST has been successfully applied in artificial intelligence, cognitive sciences, machine learning, knowledge acquisition, decision analysis, knowledge discovery from databases, and so on [3, 10, 12]. The equivalence relation is the mathematical basis for RST. The set of all equivalence objects is called equivalence class, and the family of equivalence classes forms a partition of the universe. A rough set is the approximation of a vague concept by a pair of precise concepts which are known as lower and upper approximations. The lower approximation is a definition of the domain objects which are known as absolutly belonging to the concept of interest (set X), while the upper approximation is the set of those objects which possibly belong to the concept of interest. The boundary region or region of uncertainty is the difference between the lower and upper approximations. By using the concept of lower and upper approximations in RST, knowledge hidden in information systems may be revealed and expressed in the form of decision rules.

However, the original rough set model is quite sensitive to noisy data. Thus, Ziarko [20] proposed the VPRS model to deal with noisy data and uncertain information. VPRS model is an extension of the classical RST as a tool for classification of objects. VPRS deals with partial classification by introducing a probability value β. The β represents a bound on the conditional probability of a proportion of objects in a condition class which are classified into the same decision class. Ziarko [20, 21] considered β as a classification error, defined to be in the interval $[0, 0.5)$ and his model degenerates into the classical rough set model if $\beta = 0$. However, An et al. [1] used the symbol β to denote the proportion of correct classification, in which case the appropriate range is $(0.5, 1]$ and their model degenerates into classical rough set model if $\beta = 1$. For VPRS models, some researchers have studied it and got some meaningful results [2, 9, 14].

A basic concept related to RST is information system (attribute-value system). Most applications based on RST can fall into the attribute-value representation model [18]. Information systems can be classified into two categories: complete and incomplete. A complete information system is a system in which the values of all the attributes are given. An incomplete information system means a system where the values of some of the attributes are not known, i.e., missing or partially known. Missing attribute values commonly exist in real world data sets. They may come from the data collecting process or redundant diagnose tests, unknown data and so on. Mining a database with incomplete data, the patterns of missing data as well as the potential implication of these missing data constitute valuable knowledge [16]. The basic idea of RST is knowledge acquisition in the sense of unraveling a set of decision rules from an information system via an objective knowledge reduction process for decision making. Various approaches using RST and VPRS have been proposed to induce decision rules from data sets taking the form of complete information systems [3, 5, 9, 11, 12, 18, 19, 20].

Due to the existence of incomplete information systems in real life, many authors have extended rough set model into incomplete information systems [4, 6, 7, 15, 16, 17]. Especially, in [15], the VPRS approaches for dealing with incomplete information system have been discussed, and a cumulative variable precision rough set model was established in order to overcome no monotonic property of the lower approximation. The purpose of this paper is to combine the VPRS model and incomplete information system, propose VPRS model for an incomplete information system different from [15]. At the same time, we present an approach to determine the parameter β which is a classification error. Then, by calculating discernibility matrix based on β-lower approximation, the β-reducts and the generalized decision rules are obtained.

To facilitate our discussion, we first present basic notions of VPRS model and incomplete information system in Section 2. The β-lower and β-upper approximations for incomplete information system are then defined in Section 3. Discernibility matrix and discernibility functions for incomplete decision table are given in Section 4. An illustrative example is analyzed in Section 5 to show the feasibility of the proposed approach. Results and comparison are summarized in Section 6.

2 PRELIMINARIES

In original rough set model, let U be a non-empty and finite set called the universe, R be an equivalence relation on U. The pair (U, R) is called the approximation space (it is also called Pawlak approximation space). The quotient set of U by the relation R is denoted by U/R, and $U/R = \{E_1, E_2, \cdots, E_m\}$, where $E_i (i \in \{1, 2, \ldots, m\})$ is equivalence class of R. Elements in the same equivalence class are said to be indistinguishable, and the equivalence classes of R are called elementary sets.

The original rough set model is quite sensitive to noisy data. When noisy data exists, the lower and the upper approximations cannot normally be formed. Ziarko [20] thus modified the original rough set model and proposed the VPRS model to solve this problem. VPRS model was aimed at handling uncertain and noisy information and was directly derived from the original rough set model without any additional assumptions allowing for some degree of misclassification in the mining process.

Let two sets X and Y be non-empty subsets of the universe U. The relative degree of misclassification of the set X with respect to set Y is defined as

$$\mathrm{c}(X, Y) = \begin{cases} 1 - \frac{|X \cap Y|}{|X|}, & |X| > 0 \\ 0, & |X| = 0 \end{cases}$$

where $|X|$ is the cardinality of X. It is clear that $0 \leq \mathrm{c}(X, Y) \leq 1$.

Based on the relative degree of misclassification, Ziarko generalized the lower and upper approximations of the original rough set model with a majority inclusion threshold β.

Definition 1 ([20])**.** Let (U, R) be an approximation space. For any $X \subseteq U$ and the parameter β $(0 \leq \beta < 0.5)$, the β-lower and the β-upper approximations of X

with respect to R are defined as follows:

$$\begin{aligned}
\underline{R}_\beta X &= \bigcup\{E \in U/R | \mathrm{c}(E,X) \le \beta\}\,;\\
\overline{R}_\beta X &= \bigcup\{E \in U/R | \mathrm{c}(E,X) < 1-\beta\}\,.
\end{aligned}$$

The β-lower approximation of X is also called the β-positive region of X, denoted as $\mathrm{POS}_\beta(X)$.

In addition, the β-boundary and the β-negative region of X with respect to R are defined as follows:

$$\begin{aligned}
\mathrm{BN}R_\beta X &= \bigcup\{E \in U/R | \beta < \mathrm{c}(E,X) < 1-\beta\};\\
\mathrm{NEG}R_\beta X &= \bigcup\{E \in U/R | \mathrm{c}(E,X) \ge 1-\beta\}.
\end{aligned}$$

Example 1. Let (U,R) be an approximation space, $U = \{x_1, x_2, \cdots, x_{20}\}$ and

$$U/R = \{E_1, E_2, E_3, E_4, E_5, E_6\},$$

where

$$\begin{aligned}
E_1 &= \{x_1, x_2, x_3, x_4, x_5\},\\
E_2 &= \{x_6, x_7, x_8\},\\
E_3 &= \{x_9, x_{10}, x_{11}, x_{12}\},\\
E_4 &= \{x_{13}, x_{14}\},\\
E_5 &= \{x_{15}, x_{16}, x_{17}, x_{18}\},\\
E_6 &= \{x_{19}, x_{20}\}.
\end{aligned}$$

For $X = \{x_4, x_5, x_8, x_{14}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}\}$, we suppose that $\beta = 0.25$, then we have

$$\begin{aligned}
c(E_1,X) &= 1 - \frac{|E_1 \cap X|}{|E_1|} = 1 - \frac{2}{5} = 0.6 < 1-\beta;\\
c(E_2,X) &= 1 - \frac{|E_2 \cap X|}{|E_2|} = 1 - \frac{1}{3} = 0.67 < 1-\beta;\\
c(E_3,X) &= 1 - \frac{|E_3 \cap X|}{|E_3|} = 1 - \frac{0}{4} = 1 > 1-\beta;\\
c(E_4,X) &= 1 - \frac{|E_4 \cap X|}{|E_4|} = 1 - \frac{1}{2} = 0.5 < 1-\beta;\\
c(E_5,X) &= 1 - \frac{|E_5 \cap X|}{|E_5|} = 1 - \frac{3}{4} = 0.25 = \beta;\\
c(E_6,X) &= 1 - \frac{|E_6 \cap X|}{|E_6|} = 1 - \frac{2}{2} = 0 \le \beta.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\underline{R}_\beta X &= E_5 \cup E_6 = \{x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}\};\\
\overline{R}_\beta X &= E_1 \cup E_2 \cup E_4 \cup E_5 \cup E_6\\
&= \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}\};\\
\mathrm{BN}R_\beta X &= E_1 \cup E_2 \cup E_4 = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_{13}, x_{14}\};\\
\mathrm{NEG}R_\beta X &= E_3 = \{x_9, x_{10}, x_{11}, x_{12}\}.
\end{aligned}$$

The classical rough set approach, based on complete information systems, cannot be directly applied in information systems with missing attribute values; so an extension of rough sets that can deal with incomplete data presented by Kryszkiewicz is given in [7].

Definition 2 ([7])**.** Information system is an ordered quadruple $\langle U, Q, V, f\rangle$, U is a non-empty finite set of objects called the universe, Q is a non-empty finite set of attributes, V is the union of attribute domains, i.e., $V = \cup_{a\in Q} V_a$, where V_a denotes the domain of the attribute a. $f : U \times Q \to V$ is an information function which associates an unique value of each attribute with each object belonging to U, i.e., for any $a \in Q$ and $x \in U$, $f(x, a) \in V_a$.

It may happen that some attribute values for an object are missing. To indicate such a situation, a distinguished value, so-called null value, is usually assigned to those attributes. If V_a contains null value for at least one attribute $a \in Q$, then the information system is called an incomplete information system. Otherwise it is a complete information system. In this paper, we will denote null value by $'*'$.

Definition 3 ([7])**.** Let $\langle U, Q, V, f\rangle$ be an incomplete information system. For attribute subset $A \subseteq Q$, the tolerance relation R_A is defined as

$$R_A = \bigcap_{a\in A} \{(x, y) \in U \times U | a(x) = a(y) \text{ or } a(x) = * \text{ or } a(y) = *\}.$$

$S_A(x) = \{y \in U | (x, y) \in R_A\}$, $S_A(x)$ is a greatest set whose objects are possibly indiscernible with x.

$S_A(x)$ is also called the tolerance class of x and

$$U/R_A = \{S_A(x_1), S_A(x_2), \cdots, S_A(x_n)\}$$

$(x_i \in U)$ denotes the set containing the tolerance classes. Tolerance classes in U/R_A do not constitute a partition of U in general. They may be subsets/supersets of each other or may overlap. Of course, $\cup_{x\in U} S_A(x) = U$.

Example 2. An incomplete information system about descriptions of several cars is given in Table 1.

From Table 1, we have $U = \{x_1, x_2, \cdots, x_6\}$, $Q = \{\mathrm{P}, \mathrm{M}, \mathrm{S}, \mathrm{X}\}$, where $\mathrm{P}, \mathrm{M}, \mathrm{S}, \mathrm{X}$ stand for Price, Mileage, Size, Max-Speed, respectively.

Car	Price	Mileage	Size	Max-Speed
x_1	High	High	Full	Low
x_2	Low	*	Full	Low
x_3	*	*	Compact	High
x_4	High	*	Full	High
x_5	*	*	Full	High
x_6	Low	High	Full	*

Table 1. An incomplete information system

We note that

$$U/SIM(Q) = \{S_Q(x_1), S_Q(x_2), S_Q(x_3), S_Q(x_4), S_Q(x_5), S_Q(x_6)\},$$

where $S_Q(x_1) = \{x_1\}$, $S_Q(x_2) = \{x_2, x_6\}$, $S_Q(x_3) = \{x_3\}$, $S_Q(x_4) = \{x_4, x_5\}$, $S_Q(x_5) = \{x_4, x_5, x_6\}$, $S_Q(x_6) = \{x_2, x_5, x_6\}$.

It can be also observed easily that

$$U/SIM(\{P, S, X\}) = U/SIM(Q),$$

but

$$U/SIM(\{S, X\}) \neq U/SIM(Q).$$

In fact,

$$U/SIM(\{S, X\}) = \{S_A(x_1), S_A(x_2), S_A(x_3), S_A(x_4), S_A(x_5), S_A(x_6)\},$$

where $A = \{S, X\}$, $S_A(x_1) = S_A(x_2) = \{x_1, x_2, x_6\}$, $S_A(x_3) = \{x_3\}$, $S_A(x_4) = S_A(x_5) = \{x_4, x_5, x_6\}$, $S_A(x_6) = \{x_1, x_2, x_4, x_5, x_6\}$.

Formally, a set $A \subseteq Q$ is a reduct of information system iff $U/SIM(A) = U/SIM(Q)$ and for any $B \subseteq A$, $U/SIM(B) \neq U/SIM(Q)$.

That is, for the incomplete information system given by Table 1, we can find out that $\{P, S, X\}$ is its reduct.

3 THE β-LOWER AND β-UPPER APPROXIMATIONS FOR INCOMPLETE INFORMATION SYSTEM

In this section, we propose the concepts of the β-lower and β-upper approximations for incomplete information system, and discuss the β-dependency degree of VPRS model for incomplete information system and the β-reducts.

Definition 4. Let $\langle U, Q, V, f\rangle$ be an incomplete information system. For $X \subseteq U$ and $A \subseteq Q$, the β-lower and the β-upper approximations of X with respect to A are defined as follows:

$$\begin{aligned} \underline{A}_\beta X &= \{x \in U | c(S_A(x), X) \leq \beta\}, \\ \overline{A}_\beta X &= \{x \in U | c(S_A(x), X) < 1 - \beta\}. \end{aligned}$$

In addition, the β-boundary and the β-negative region of X with respect to A are defined as follows:

$$\begin{aligned}\mathrm{BN}A_\beta X &= \{x \in U | \beta < c(S_A(x), X) < 1-\beta\},\\ \mathrm{NEG}A_\beta X &= \{x \in U | c(S_A(x), X) \geq 1-\beta\}.\end{aligned}$$

If $\underline{A}_\beta X = \overline{A}_\beta X$, i.e., $\mathrm{BN}A_\beta X = \emptyset$, then X is called β-discernible with respect to A. Otherwise, X is called β-indiscernible.

Remark 1. If $\langle U, Q, V, f\rangle$ is a complete information system, then the β-lower and the β-upper approximations of X in Definition 4 would degenerate into the β-lower and the β-upper approximations in [20].

Theorem 1. Let $\langle U, Q, V, f\rangle$ be an incomplete information system. For $X \subseteq U$ and $A \subseteq Q$. With respect to A, if X is β-discernible at level $0 \leq \beta < 0.5$, then it is γ-discernible at any level $\gamma > \beta$.

Proof. With respect to A, if X is β-discernible at level $0 \leq \beta < 0.5$, then $\underline{A}_\beta X = \overline{A}_\beta X$, i.e.,

$$\{x \in U | c(S_A(x), X) \leq \beta\} = \{x \in U | c(S_A(x), X) < 1-\beta\}.$$

That is, there does not exist an $x \in U$, such that $x \in \underline{A}_\beta X$ or $x \in \overline{A}_\beta X$ when $\beta < c(S_A(x), X) < 1-\beta$.

Since for any $\gamma > \beta$, $\beta < \gamma < c(S_A(x), X) < 1-\gamma < 1-\beta$, there does not exist an $x \in U$, such that $x \in \underline{A}_\gamma X$ or $x \in \overline{A}_\gamma X$. Therefore, $\underline{A}_\gamma X = \overline{A}_\gamma X$, i.e., X is γ-discernible at level γ. □

Theorem 2. Let $\langle U, Q, V, f\rangle$ be an incomplete information system. For $X \subseteq U$ and $A \subseteq Q$. With respect to A, if X is β-indiscernible at level $0 \leq \beta < 0.5$, then it is γ-indiscernible at any level $\gamma < \beta$.

Proof. It follows from Theorem 1 directly. □

Definition 5. Let $\langle U, C \cup D, V, f\rangle$ be an incomplete information system. C is the set of condition attributes, D is a decision attribute. $A \subseteq C$. Then the β-dependency degree between A and D is defined as:

$$\gamma(A, D, \beta) = \frac{|\mathrm{POS}(A, D, \beta)|}{|U|}. \tag{1}$$

where $\mathrm{POS}(A, D, \beta) = \{x | x \in U, c(S_A(x), E) \leq \beta\}$, $E \in U/D$.

Ziarko [20] also states that if $X(X \subseteq U)$ is β-indiscernible at every level β, then X will be called absolutely rough. Otherwise, X will be called relatively rough. For every relatively rough set X, there exists a classification error level β at least such that X is β-discernible at this level. The minimum of these β is called the discernible threshold. In fact, we usually need the ranges of β where X can be discerned rather than a specific β, a central idea of this paper. While by Theorem 1 we know that if we want to get the ranges of β which X can be discerned, we only need get the discernible threshold.

In the following part, we attempt to determine the discernible threshold β.

Let $\langle U, C \cup D, V, f \rangle$ be an incomplete information system, $U = \{x_1, x_2, \cdots, x_n\}$, C be the set of condition attributes, D be a decision attribute. For any $A \subseteq C$, $S^* = \{S_A(x_1), S_A(x_2), \ldots, S_A(x_n)\}$ denotes the tolerance classes. Then with respect to A, the discernible threshold β can be calculated as follows:

$$\beta = \xi(A, X) = \max(m_1, m_2), \tag{2}$$

where

$$\begin{aligned} m_1 &= 1 - \min\{c(S_A(x), X) | c(S_A(x), X) > 0.5\}, \\ m_2 &= \max\{c(S_A(x), X) | c(S_A(x), X) < 0.5\}. \end{aligned}$$

Furthermore, the β-reducts and the generalized decision rules for an incomplete information system can be obtained according to the steps below:

Step 1. For each condition attribute $A (A \subseteq C)$, according to (1), calculate the dependency degree between A and D.

Step 2. Removes redundant attributes. A condition attribute subset $A \subseteq C$ is called a β-reduct if and only if it satisfies $\gamma(A, D, \beta) = \gamma(C, D, \beta)$ and there does not exist a condition attribute subset $B \subsetneq A$ such that $\gamma(B, D, \beta) = \gamma(C, D, \beta)$.

Step 3. Generalized decision rules are obtained according to the final β-reducts. (Generalized decision rules can be expressed as $r_{ij} : \mathrm{des}(X_i) \to \mathrm{des}(Y_j)$, where X_i expresses the description of objects in universe according to β-reducts, Y_j expresses the description of objects in universe according to decision attributes D. Formally, generalized decision rules can be expressed as $\wedge(c, v) \to \vee(d, w)$, where c is condition attribute and v is condition attribute value, d is decision attribute and w is decision attribute value.)

4 INCOMPLETE DECISION TABLE, DISCERNIBILITY MATRIX AND DISCERNIBILITY FUNCTIONS

The discernibility matrix was developed by Skowron and Rauszer [13]. An element of the matrix is the set of all attributes that distinguish the corresponding object pairs, namely, the set consists of all attributes on which the corresponding two objects have distinct values. One can construct a Boolean discernibility function from a discernibility relation, with attributes as Boolean variables. Skowron and Rauszer showed that the set of attribute reducts is in fact the set of prime implicants of the reduced disjunctive form of the discernibility function. This provides a logic foundation for the study of reducts.

In this section, we will present the discernibility matrix and discernibility functions based on the incomplete information system.

Incomplete decision table is an incomplete information system $\langle U, C \cup d, V, f\rangle$, where the elements of C are called condition attributes, $d \notin C$ and $* \notin V_d$, is a distinguished attribute called decision attribute.

Define function $\partial_A(x) : U \to P(V_d)$, $A \subseteq Q$, as follows:

$$\partial_A(x) = \{i | i = d(y), y \in S_A(x)\}.$$

It is also said to be a generalized decision function in incomplete decision table.

Definition 6 ([8])**.** Let $\langle U, Q, V, f\rangle$ be an incomplete information system. For any $x, y \in U$, $a \in Q$, the discernibility function $\delta_Q(x, y)$ between x and y is defined as follows:

$$\delta_Q(x, y) = \{a \in Q | a(x) \neq a(y) \wedge a(x) \neq * \wedge a(y) \neq *\} = \{a \in Q | (x, y) \notin R_a\}.$$

Δ is a discernibility function for incomplete information system iff

$$\Delta = \prod_{(x,y)\in U\times U} \sum \delta_Q(x, y).$$

$\Delta(x)$ is a discernibility function for object x in incomplete information system iff

$$\Delta(x) = \prod_{y\in U} \sum \delta_Q(x, y).$$

Δ^* is a discernibility function for incomplete decision table iff

$$\Delta^* = \prod_{(x,y)\in U\times\{z\in U | d(z)\notin\partial_C(x)\}} \sum \delta_C(x, y).$$

$\Delta^*(x)$ is a discernibility function for object x in incomplete decision table iff

$$\Delta^*(x) = \prod_{y\in\{z\in U | d(z)\notin\partial_C(x)\}} \sum \delta_C(x, y).$$

(Where $\prod$ and $\sum$ express conjunction and disjunction, respectively.)

Discernibility matrix can be used to find the minimal subset(s) of attributes, which leads to the same partition of the data as the whole set of attributes Q. To do this, first we have to construct discernibility function. This is a Boolean function. The core is the set of all the single element in the discernibility matrix, defined as follows:

$$core(Q) = \{a \in Q | \delta_Q(x, y) = \{a\}\}$$

where $x, y \in U$.

5 A CASE STUDY

In this section, an example is given to show the validity and practicability of β-reducts which is proposed in this paper.

Example 3. Given descriptions of several cars as in Table 2 [8]. Let us try to classify them according to the chosen subsets of attributes. From Table 2, $U = \{1, 2, 3, 4, 5, 6\}$. $Q = \{\mathrm{P}, \mathrm{M}, \mathrm{S}, \mathrm{X}\}$, where P, M, S, X stands for Price, Mileage, Size, Max-Speed, respectively.

Car	Price	Mileage	Size	Max-Speed	D
1	High	High	Full	Low	Good
2	Low	*	Full	Low	Good
3	*	*	Compact	High	Poor
4	High	*	Full	High	Good
5	*	*	Full	High	Excel
6	Low	High	Full	*	Good

Table 2. An incomplete information system

1. Tolerance classes and decision classes.

   Tolerance classes can be calculated as follows:

   $$U/R_Q = \{S_Q(1), S_Q(2), S_Q(3), S_Q(4), S_Q(5), S_Q(6)\}$$

   where $S_Q(1) = \{1\}$, $S_Q(2) = \{2, 6\}$, $S_Q(3) = \{3\}$, $S_Q(4) = \{4, 5\}$, $S_Q(5) = \{4, 5, 6\}$, $S_Q(6) = \{2, 5, 6\}$.

   Decision classes can also be obtained as follows: $U/D = \{D_{Good}, D_{Poor}, D_{Excel}\}$, $D_{Good} = \{1, 2, 4, 6\}$, $D_{Poor} = \{3\}$, $D_{Excel} = \{5\}$.

2. Determine the precision parameter β

   By the formula (2),

   $$\begin{aligned} \xi(Q, D) &= \max(m_1, m_2). \\ m_1 &= 1 - \min\{c(S_Q(x), E) | c(S_Q(x), E) > 0.5\}, \\ m_2 &= \max\{c(S_Q(x), E) | c(S_Q(x), E) < 0.5\}. \end{aligned}$$

   where E denotes the equivalence classes based on decision attribute D, and

   $$U/D = \{D_{Good}, D_{Poor}, D_{Excel}\}.$$

   Since

   $$c(S_Q(1), D_{Good}) = 1 - \frac{1}{1} = 0,$$

$$
\begin{aligned}
c(S_Q(1), D_{Poor}) &= 1 - \frac{0}{1} = 1, \\
c(S_Q(1), D_{Excel}) &= 1 - \frac{0}{1} = 1, \\
c(S_Q(2), D_{Good}) &= 1 - \frac{2}{2} = 0, \\
c(S_Q(2), D_{Poor}) &= 1 - \frac{0}{1} = 1, \\
c(S_Q(2), D_{Excel}) &= 1 - \frac{0}{1} = 1, \\
c(S_Q(3), D_{Good}) &= 1 - \frac{0}{1} = 1, \\
c(S_Q(3), D_{Poor}) &= 1 - \frac{1}{1} = 0, \\
c(S_Q(3), D_{Excel}) &= 1 - \frac{0}{1} = 1, \\
c(S_Q(4), D_{Good}) &= 1 - \frac{1}{2} = \frac{1}{2}, \\
c(S_Q(4), D_{Poor}) &= 1 - \frac{0}{1} = 1, \\
c(S_Q(4), D_{Excel}) &= 1 - \frac{1}{2} = \frac{1}{2}, \\
c(S_Q(5), D_{Good}) &= 1 - \frac{2}{3} = \frac{1}{3}, \\
c(S_Q(5), D_{Poor}) &= 1 - \frac{0}{1} = 1, \\
c(S_Q(5), D_{Excel}) &= 1 - \frac{1}{3} = \frac{2}{3}, \\
c(S_Q(6), D_{Good}) &= 1 - \frac{2}{3} = \frac{1}{3}, \\
c(S_Q(6), D_{Poor}) &= 1 - \frac{0}{1} = 1, \\
c(S_Q(6), D_{Excel}) &= 1 - \frac{1}{3} = \frac{2}{3}.
\end{aligned}
$$

Therefore,

$$m_1 = 1 - \min\left(1, \frac{2}{3}\right) = \frac{1}{3}, m_2 = \max\left(0, \frac{1}{3}\right) = \frac{1}{3}.$$

Thus,

$$\xi(Q, D) = \max(m_1, m_2) = \frac{1}{3}.$$

That is, we have the precision parameter $\beta = \frac{1}{3}$.

3. The set of the β-reducts

For $\beta = \frac{1}{3}$, $\underline{Q}_\beta(D_{Good}) = \{1, 2, 5, 6\}$, $\underline{Q}_\beta(D_{Poor}) = \{3\}$, $\underline{Q}_\beta(D_{Excel}) = \{\emptyset\}$.

	1	2	3	5	6
1	-	-	S, X	X	-
2	-	-	S, X	X	-
3	S, X	S, X	-	S	S
5	-	-	S	-	-
6	-	-	S	-	-

Table 3. The discernibility matrix for incomplete decision table

The discernibility functions Δ^* are given as follows:

$$\begin{aligned}
\Delta^*_\beta(1) &= (S \vee X) \wedge X = X, \\
\Delta^*_\beta(2) &= (S \vee X) \wedge X = X, \\
\Delta^*_\beta(3) &= (S \vee X) \wedge S = S, \\
\Delta^*_\beta(5) &= S, \\
\Delta^*_\beta(6) &= S, \\
\Delta^*_\beta(D_1) &= \Delta^*_\beta(1) * \Delta^*_\beta(2) * \Delta^*_\beta(5) * \Delta^*_\beta(6) = X \wedge X \wedge S \wedge S = X \wedge S, \\
\Delta^*_\beta(D_2) &= \Delta^*_\beta(3) = S, \\
\Delta^*_\beta(D) &= \Delta^*_\beta(D_1) * \Delta^*_\beta(D_2) = X \wedge S \wedge S = X \wedge S.
\end{aligned}$$

So $\{\mathrm{X}, \mathrm{S}\}$ is the β-reducts for incomplete decision table, the incomplete decision table for β-reducts $\{\mathrm{X}, \mathrm{S}\}$ is presented in Table 4.

Car	Size	Max-Speed	D
1	Full	Low	Good
2	Full	Low	Good
3	Compact	High	Poor
5	Full	High	Excel
6	Full	*	Good

Table 4. The incomplete decision table for β-reducts

We are interested in decision rules for β-reducts. Thus, Table 5 presents the incomplete discernibility matrix for the β-reducts $\{\mathrm{X}, \mathrm{S}\}$.

	1	2	3	5	6
1	-	-	S, X	X	-
2	-	-	S, X	X	-
3	S, X	S, X	-	S	S
5	-	-	S	-	-
6	-	-	S	-	-

Table 5. The incomplete discernibility matrix for the β-reducts

The relative discernibility function is given as follows:

$$\begin{aligned}\Delta^*(1) &= (S \vee X) \wedge X = X,\\ \Delta^*(2) &= (S \vee X) \wedge X = X,\\ \Delta^*(3) &= (S \vee X) \wedge S = S,\\ \Delta^*(5) &= S,\\ \Delta^*(6) &= S.\end{aligned}$$

4. Generalized decision rules

Car	Size	Max-Speed	D
1	-	Low	Good
2	-	Low	Good
3	Compact	-	Poor
5	Full	-	Excel
6	Full	-	Good

Table 6. The final version in the subset $\{\mathrm{S},\mathrm{X}\}$ of incomplete decision table

According to Table 6, the generalized decision rules can be obtained as follows:

$$\begin{aligned}r_1 &: (\mathrm{X},\mathrm{Low}) \to (\mathrm{D},\mathrm{Good});\\ r_2 &: (\mathrm{S},\mathrm{Full}) \to (\mathrm{D},\mathrm{Good}) \vee (\mathrm{D},\mathrm{Excel});\\ r_3 &: (\mathrm{S},\mathrm{Compact}) \to (\mathrm{D},\mathrm{Poor});\end{aligned}$$

6 CONCLUSIONS

The study of reducts is fundamental in rough set theory. The concept of a discernibility matrix enables us to establish a logical and theoretical foundation for reducts of an information table. In this paper, we combine the variable precision rough set (VPRS) model and incomplete information system, the variable precision rough set model for an incomplete information system is proposed. At the same time, we present an approach to determine the parameter β and obtained the β-reducts using the discernibility matrix. We compared our method with the approach in [8], and find the decision rules which we obtained is the same as the conclusion in [8]; but we use classification error $\beta = \frac{1}{3}$, such that the lower and upper approximations of X are increased, then it has adaptive faculty for noise data. On the other hand, we use discernibility function and discernibility matrix to obtain the β-reducts, which simplified the process of calculate.

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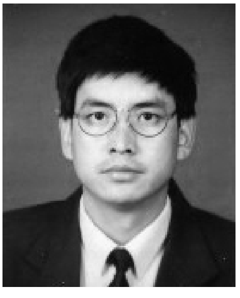

Zengtai Gong received his Ph. D. degree in science from Harbin Institute of Technology in China. He is a Professor and Doctor Supervisor at the College of Mathematics and Statistics, Northwest Normal University in China. His research topics include rough set theory, fuzzy analysis, real analysis, and their applications in environmental problems and management of water resources.

Zhanhong Shi received his Master degree in science from Northwest Normal University in China. He is a lecturer at College of Science of Gansu Agricultural University in China; currently he is working towards his Ph. D. degree at the College of Mathematics and Statistics, Northwest Normal University in China. His research topics include rough set theory and decision making.

Hongxia Yao received her Master degree in science from Northwest Normal University in China. She is a bank clerk in Guangzhou, China. Her research topics include rough set theory and information processing.