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NEW MODEL OF MAXIMAL COVERING LOCATION PROBLEM WITH FUZZY CONDITIONS

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Abstract. The objective of Maximal Covering Location Problem is locating facilities such that they cover the maximal number of locations in a given radius or travel time. MCLP is applied in many different real-world problems with several modifications. In this paper a new model of MCLP with fuzzy conditions is presented. It uses two types of fuzzy numbers for describing two main parameters of MCLP – coverage radius and distances between locations. First, the model is defined, then Particle Swarm Optimization method for solving the problem is described and tested.

Keywords: Maximal Covering Location Problem (MCLP), fuzzy conditions, Particle Swarm Optimization (PSO)

Mathematics Subject Classification 2010: 93A30, 90C27, 90C59

1 INTRODUCTION

Location problems represent a very important class of optimization problems with applications in various branches of science, industry and real-life problems. Covering location problem is a very well known location problem and it has been well studied in the past. Objective of the covering location problem is to find the best locations for various facilities, as shops, schools, emergency units, antennas, pollution sources, etc. There are three different types of location problems:

- Location set covering problem (LSCP), with the aim to cover all locations with the least number of facilities,
- Maximal covering location problem (MCLP), with the aim to cover maximal number of locations with fixed number of facilities and
- Minimal covering location problem (MinCLP), with the aim to cover minimal number of locations with fixed number of facilities and given minimal distance between each facility.

The MCLP was introduced by Church and ReVelle in 1974 [1] on network, and since then, many variants of it have been proposed. Original MCLP uses previously given fixed values for covering radius or travel time, but in modeling real problems this approach is not completely accurate. Real problems contain some degree of uncertainty, for example covering radius is about 5 kilometers or travel time is between 8 and 10 minutes. In order to model problems with indefinite conditions fuzzy variables are used. Two models with fuzzy coverage radius have been described by Davari et al. in [2] and [3] and this modification of MCLP is known as fuzzy maximum location problem – FMCLP. Proposed fuzzy methods do not study locations that are partially covered with several facilities. There are two different classes of problems used to describe a partial coverage. First class consists of problems where it is allowed to sum covering degrees of partially covered locations. For example, each source of light has a radius of full illumination and another external radius where illumination decreases. For locations that are in two or more external radii, total lighting is calculated as the sum of all partial illuminations. The second class consists of problems where it is not possible to sum the degrees of partial coverages. There are a lot of problems of this type – all MCLP problems regarding buildings location belong to this class. It means that if some location is partially covered by more facilities, quality of the solution will not increase. For example, each building (shop, emergency unit …) has a main radius of coverage and all locations in the main radius have full coverage. The second parameter is the external radius, where the degree of coverage decreases. If some location is partially covered by more facilities, it must be associated with the nearest location.

The aim of this paper is to present a new model for maximal covering location problem which associates each partially covered location to its nearest facility and a new method for modeling FMCLP with simultaneous fuzzyfication of two MCLP key conditions – covering radius (or travel time) and distances between locations.

Many methods for solving MCLP and FMCLP have been developed and described in the literature. Some of them are exact methods, but they are inapplicable for problems with large dimensions, so a lot of heuristic and metaheuristic methods for solving MCLP and FMCLP have evolved. The most present methods are Genetic Algorithms, Tabu Search, VNS and Simulated Annealing method which will be discussed in the next section. In this study we use a modification of Particle Swarm Optimization method (PSO) for solving the proposed model.

The rest of the paper is organized as follows: First, in Section 2, there is a brief review of literature about MCLP and FMCLP and a review of methods used to solve them. In Section 3 a detailed description of the problem with an illustrative example and mathematical model is given. Used PSO method for solving a proposed model is presented in Section 4, and in Section 5 numerical tests are given. Finally, in Section 6, conclusion with ideas for further research is presented.

2 LITERATURE REVIEW

Some papers with key ideas that illustrate the evolution of MCLP, FMCLP and methods for solving them will be shown in the next paragraph. Detailed introduction to location problems is found in [4, 5, 6, 7, 8].

As mentioned before, the first description of MCLP was given by Church and ReVelle in 1974 [1] and since then a lot of modifications, extensions and adaptations of MCLP have been introduced. Moore and Revelle studied the hierarchical service location problem in [9], gradual covering problem is described by Berman and Krass in [10] and Qu and Weng studied the problem of multiple allocation hub maximal covering problem [11]. MCLP is originally defined on network with travel times between each pair of location, but some authors defined MCLP as a problem of covering points on the plane with given Euclidian distances between them [12]. Several papers are dedicated to application of MCLP for finding solutions of real-life problems, like maximal covering model for network design [13], model for determining the distribution of police patrol areas in Dallas [15], models for improving accessibility to rural health services in Ghana [14] and model for locating emergency medical services in Istanbul [16].

Originally, MCLP is defined in a deterministic sense, but in later studies authors describe stochastic and fuzzy models for MCLP with uncertainty parameters. Louveaux in [8] and Weaver and Church in [17] defined uncertainty as a probability distribution of some input parameters. On the other hand, some authors introduced vagueness in MCLP model using fuzzy sets. Perez et al. in [19] described a fuzzy model for real-world problems with linguistic vagueness. Darzentas introduced in [18] a model for covering based on fuzzy set partitioning. Batanovic et al. [20] described one more application of fuzzy sets in modeling maximum covering location problems in networks in uncertain environments and Davari et al. in [2] presented an MCLP model with fuzzy variables for travel times for any pairs of nodes.

A lot of methods for solving MCLP and FMCLP have been presented in literature. Downs and Camm presented an exact algorithm for MCLP using dual-based solution method and greedy heuristic [21]. Some authors used VNS method [3], simulated annealing [12], genetic algorithms [23] and some hybrid algorithms for solving facility location problem. Detailed review of literature related to these methods is found in [22].

3 A NEW FORMULATION OF FMCLP

As mentioned before, the main goal of this study is defining a new model of FMCLP for better uncertainty modeling in real-life problems. In this study, all nodes have the same importance – population in locations is not considered. Also, in this paper terms radius of coverage and travel time present the same variable which determine degree of coverage of locations.

First, uncertainty is represented as fuzzy radius of coverage (fuzzy travel time) and it depends on two parameters – main radius R represents radius of full coverage and external radius f_r represents a radius of partial coverage. If the distance between locations is less than the main radius, it is fully covered and its degree of coverage is equal to 1. If location is between main and external radii, its degree of coverage is value between 0 and 1, and if the location is outside of the external radius it is not covered and its degree is 0.

Motivation for introduction fuzzy radius of coverage lies in the nature of covering location problems. In a standard model of MCLP, locations close to radius of coverage do not have any effect on solution's quality. That approach is not precise in modeling of real-life problems because informations about these locations are very important – solution is better if it contains as more as possible locations close to radius of coverage. Figures 1 and 2 illustrate the optimal solution for standard MCLP model and the optimal solution for the proposed FMCLP model on the same set of locations. MCLP found an optimal solution with 9 fully covered locations and FMCLP found an optimal solution with 7 fully covered locations and 6 partially covered locations. In the solution of MCLP three non-covered locations are very far from coverage border, but in the solution of FMCLP not fully-covered location are very close to coverage border. In this example, one location (L_1) is covered by both external radii, and this opens up the possibility to additional research related to methods for treatment multiply partially covered location. This will be discussed at the end of this section.

A problem of locating ambulances in given set of locations is an illustrative example. Here, distances between locations, main and external radii are known. The emergency service has the full effect on locations inside the main circle of coverage and some partial effect on locations between main and external radius. For example, main radius of an ambulance coverage could be 5 minutes – it means that the patient will surely survive if an ambulance can reach him or her in less than 5 minutes. External radius could be 10 minutes – it means if an ambu-

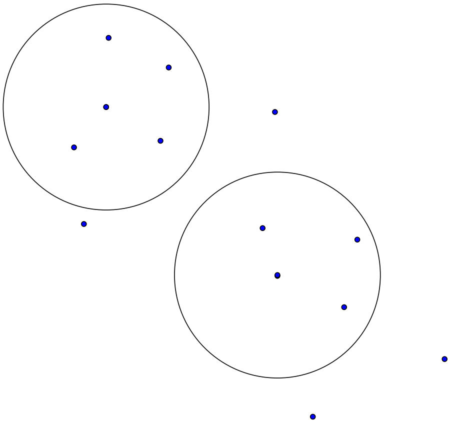

Figure 1. Optimal solution of MCLP

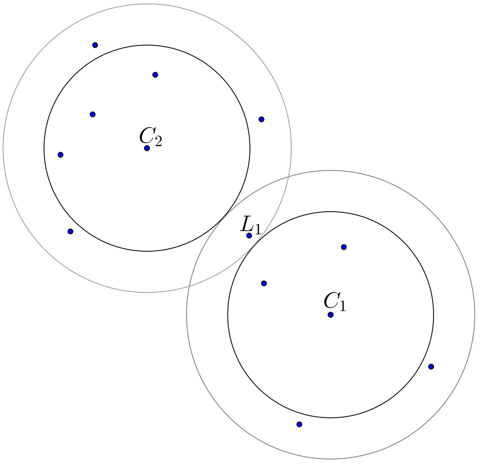

Figure 2. Optimal solution of FMCLP

lance arrives between 5 and 15 minutes, patient has a chance to survive, and if an ambulance arrives in more that 15 minutes, chances for survival are minimal.

The second uncertainty of the proposed model is the distance (or travel time) between locations. In previously described example, distances between two locations could not be precisely defined. Many external conditions influence the distance, like traffic jam, traffic light and speed of emergency vehicles. For this reason, distance (travel time) between two locations has an approximative value – 5 minutes plus-minus 1 minute.

Fuzzy radius is defined as a right shoulder fuzzy number and triangular fuzzy number is used to represent fuzzy distance value, and degree of coverage locations is defined as intersection of these fuzzy numbers. Figure 3 illustrates the formula for calculating a degree of coverage for each location. In this figure, x-axe represent distance (or travel times) between location and y-axe represents the location's degree of coverage. Intersection of right shoulder fuzzy number and triangular fuzzy number is defined as arithmetic mean of fuzzy values of intersection points S_1 and S_2.

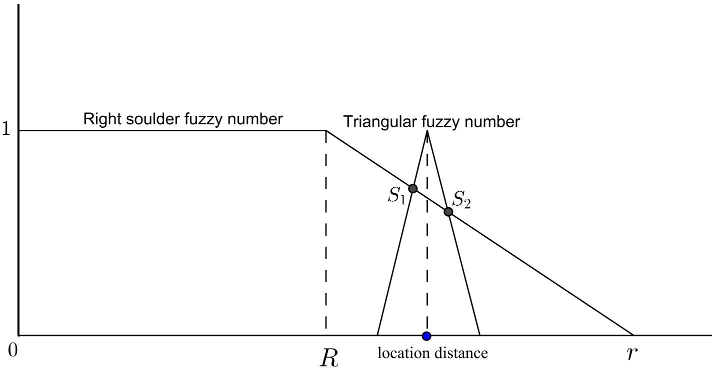

Figure 3. Intersection of right shoulder fuzzy number which represents a fuzzy radius and triangular fuzzy number which represents fuzzy number of distance between locations

As mentioned before, in the described formulation of MCLP, the open question is how to treat locations which are partially covered with two or more facilities. The first method is to compute limited sum of all degrees of coverage for each partially covered location. Sum must be limited because its degree cannot be greater than 1. This approach could be applied if location could be assigned to more facilities. An obvious example of this type is light covering location problem where the objective is to locate illumination sources in space – total illumination is equal to sum of illumination from all sources. But, this method is not applicable on most real-life maximal covering location problems – the described example of finding locations for ambulances is a good example of that if some location is partially covered with several hospitals. An addition of degrees of coverage does not make sense because each location must be assigned to exactly one facility. It is clear that assignment to the nearest facility gives the best result.

Illustration of difference between these methods is given in Figures 4 and 5. Figures show an instance with 6 locations and objective to find optimal locations for 2 facilities. Figure 4 presents an optimal solution for FMCLP with limited sum method and Figure 5 presents an optimal solution for FMCLP with maximal degree method on the same instance. For the first method total coverage degree is 3.2 – two locations are fully covered (locations of facilities C_1 and C_2), location L_1 is partially covered by two fuzzy radii with degree 0.5 and locations L_2 and L_3 are covered with degree of 0.1. If solution is recalculated using maximal degree method, results is 2.7 – location L_1 now has degree 0.5. Figure 5 shows that result 2.7 is not optimal for maximal degree method – optimal result is 2.8 – locations of facilities C_1 and C_2 are fully covered, location L_1 is covered by degree 0.5 and other locations is covered by degree of 0.1.

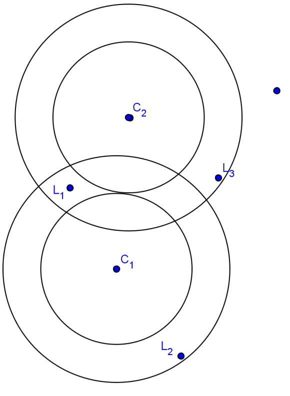

Figure 4. FMCLP with limited sum method

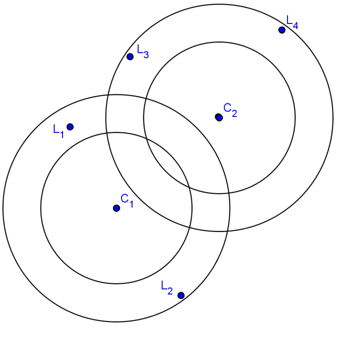

Figure 5. FMCLP with maximal degree method

The most of real-life problems belong to class of FMCLP problems with maximal degree method. This method has not been seriously considered in the past, but the proposed FMCLP model uses this method.

3.1 Mathematical Model

Mathematical model of the proposed FMCLP represents an extension of original model of MCLP given by Church and ReVelle in [1]. Innovations in this model are: a new method for computation degrees of coverage and a new condition for assignment partial covered location to the nearest center.

First, it is necessary to introduce the problem parameters and the decision variables. The following notation is given:

- I – set of all locations,
- J – set of all facility sites,
- $R + f_r$ – fuzzy radius of coverage [right shoulder fuzzy number],
- $d_{ij} \pm f_t$ – fuzzy distance (travel time) between locations i and j [triangular fuzzy number],
- $x_j = \begin{cases} 1, & \text{if facility allocated in location } j, \\ 0, & \text{otherwise.} \end{cases}$
- $y_i \in [0, 1]$ – degree of coverage of location i,
- c_{ij} – degree of coverage of location i by facility located in site j, given by formula

$$x_j = \begin{cases} 1, & d_{ij} <= R, \\ 0, & d_{ij} > R + f_r, \\ \frac{1}{2}(\mu(v_1) + \mu(v_2)), & \text{otherwise.} \end{cases}$$

  where $\mu(v_i)$ represents an intersection of fuzzy values $R + f_r$ and $d_{ij} \pm f_t$.
- p – number of facilities to be located.

Mathematical model of FMCLP is defined as follows:

$$\max \sum y_i \tag{1}$$

with conditions:

$$\sum x_i = p, \tag{2}$$

$$\max x_j \cdot c_{ij} \geq y_i, \forall j, \tag{3}$$

$$x_i \in \{0, 1\}, \tag{4}$$

$$y_i \in [0, 1]. \tag{5}$$

The objective of FMCLP is the maximization of target function (1). Condition (2) provides that solution has exactly p facilities. Constraint (3) provides that location i reaches its maximal degree of coverage and constraints (4) and (5) determine the ranges of described variables x_i and y_i.

4 PSO METHOD

In this study, Particle Swarm Optimization method (PSO) is used for solving the proposed FMCLP model. Particle swarm optimization is a well-known nature-based metaheuristic method, introduced by Kennedy and Eberhart in [24, 25]. This method is inspired by social behaviour of particles in swarms, like birds in flocks. The main idea is to create several solution instances (particles) that are moving through the solution space with some given intelligence. Each particle knows its best position so far and the best position of its neighbourhood, and updates its own position using this information. Original PSO method is developed for problems with continuous variables, but Kennedy and Eberhart developed a discrete version of this method [26], known as Discrete Particle Swarm Optimization (DPSO).

DPSO considers a swarm size S containing n particles. Each particle is represented by its position in d-dimensional binary solution space as vector $x_i = (x_{i1}, x_{i2}, \ldots, x_{id})$. Each particle has its own velocity vector $v_i = (v_{i1}, v_{i2}, \ldots, v_{id})$ in d-dimensional continuous space $\mathbf{R}^d$. Vector x_i represents a solution of the problem, while the velocity vector represents a change of particle position in the next iteration. With given notation, position of particle i in k^{th} iteration is $x_i^k = x_i^{k-1} + v_i^k$.

As mentioned before, particle's velocity determines its position in the next iteration and it depends on two parameters: its best position so far (b_i) and the best position (c_i) of its neighbourhood $N(i) \subseteq S$. Velocity vector is calculated by formula given by Kennedy and Eberhart in [24]:

$$v_i^k = v_i^{k-1} + c_1\xi_1(b_i - x_i^{k-1}) + c_2\xi_2(g_i - x_i^{k-1}) \tag{6}$$

Parameters c_1 and c_2 represent degrees of confidence of particle i in the different positions that influence its dynamics. Parameters ξ_1 and ξ_2 are independent random variables generated in each iteration with uniform distributions in the interval $[0, 1]$.

Velocity values are limited, i.e.

$$|v_{ij}| < V_{max} \tag{7}$$

where V_{max} usually has value close to 6.0. This setting prevents that the probability is either too low or too high.

In DPSO, velocity vector refers to a probability that the j^{th} binary variable in the i^{th} assumption obtains a value 0 or 1 in the next iteration. Each position variable x_{ij} of particle i gets value 1 if a randomly generated value is less than the sigmoid function:

$$\frac{1}{1 + \exp(-v_{ij})}. \tag{8}$$

PSO and DPSO were used for solving many location problems (see [27]), some hybrid modifications of PSO are presented in [28] and some models with fuzzy random uncertainty are solved in [29].

In the coding of DPSO algorithm for proposed FMCLP model, swarm contains 10 particles. Each particle in swarm has a position vector, velocity vector and vectors for storing its best position so far and the best global position. Position vectors are composed of binary values and each binary value represents a facility location, i.e. value 1 if the facility established in it, and 0 otherwise.

The algorithm starts with generating random position vectors and random velocity vectors for each particle. In the initial position vector locations for facilities are randomly chosen. Values in velocity vectors are randomly selected from the interval $[-V_{max}, V_{max}]$, where $V_{max} = 6$. In each iteration, particle position is updated by the probability method given by formula (8) and velocity is updated using (6). After each calculation of a new position of the particle, algorithm performs corrections if the particle contains the incorrect number of positions for facilities. If there are less positions, randomly selected locations are added to the solution vector and if there are more potential positions, randomly selected locations are removed.

Algorithm's input data are n locations with coordinates, number of facilities p, fuzzy coverage radius $R + f_r$ and fuzzy distances between locations $d_{ij} + f_t$. Distances between locations are calculated as Euclidean distances. Output data of algorithm is the value of the goal function (1) and the solution vector.

Algorithm: PSO for FMCLP
Input: Locations with distances, number of facilities, fuzzy radius of coverage, fuzzy distance value, maximal number of iteration
Output: Best position for distances and degree of coverage

```
for each particle do
    Init particle's position and velocity vectors with random values
    Init particle's best position
endfor
Calculate global best position
while not maxiteration do
    for each particle do
        update particle's position and velocity
        compute particle's result (degree of locations' coverage)
        if current result > best particle's result do
            particle's best result = current result
        endif
        if current result > global best result do
            global best result = current result
            solution = postion of current particle
        endif
   endwhile
endwhile
```

Figures 6 and 7 describe two visualized solutions. The algorithm has been executed twice on the same instance with different input parameters. Locations are positioned in 30×30 grid and algorithm searches for the best positions of 10 facilities with a coverage radius $R = 5$. In the first run, input data does not contain fuzzy conditions $f_r = f_t = 0$ and in the second run, fuzzy variables $f_r = 0.5$ and $f_t = 0.1$ are defined in input data. Figure 7 presents fuzzy radius as an external circle. The solution without fuzzy conditions covers 84 locations while 85.506 locations are covered with fuzzy conditions in the input data.

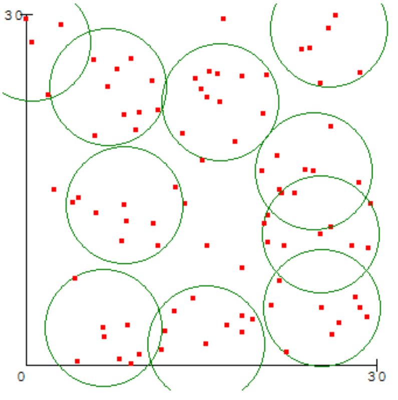

Figure 6. Solution for problem with input parameters $n = 90$, $p = 10$, $R + f_r = 5 + 0$, $f_t = 0$

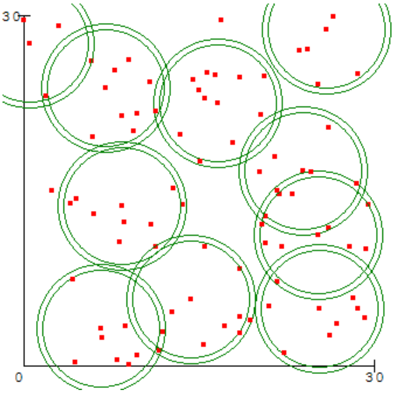

Figure 7. Solution for problem with input parameters $n = 90$, $p = 10$, $R + f_r = 5 + 0.5$, $f_t = 0.1$

5 COMPUTATIONAL RESULTS

Computational tests were performed on the generated instances of the problem. The same procedure as in ReVelle et al. [30] is used – locations are randomly set in

30×30 grid, without the associated population data – each location has the same importance.

Two algorithms are coded – the first one uses IBM CPLEX optimizier v12.1 for solving proposed model, and the second one uses the described PSO method. Both algorithms are coded in Visual C# .NET 2010 and all tests were running on the computer with Intel Core i7 800 2.8 GHz processor with 8 GB RAM memory and Windows 7 Professional operating system. In all executions, swarms in the PSO method were composed of 10 particles. Testing procedure was as follows: For each instance the algorithm runs 10 times, and the best result and the best reach time were taken. The algorithm stops if the result is unchanged after 2 000 iterations.

In the first test, results of the developed metaheuristic are compared with the results obtained using the CPLEX optimizer. Because CPLEX could successfully solve just instances with relatively small dimensions, problem dimensions are chosen so that CPLEX could solve it. In the first test, both algorithms are performed on the same instances of dimensions with 70, 80 and 90 locations. Other problem parameters are: fuzzy radii $R + f_r = 5 + 0.5, 5 + 1$, fuzzy distances $f_t = 0.1, 0.2$ and number of facilities $P = 10$. Results of the first test are shown in Table 1.

			CPLEX		PSO		
n	$R + f_r$	f_t	Solution	Time (ms)	Solution	Time (ms)	GAP (%)
70	$5 + 0.5$	0.1	68.018	38 794	68.018	259	0
70	$5 + 0.5$	0.2	68.087	34 786	68.087	376	0
70	$5 + 1$	0.1	68.416	112 369	68.416	202	0
70	$5 + 1$	0.2	68.398	112 369	68.398	418	0
80	$5 + 0.5$	0.1	76.811	3 415 977	76.811	276	0
80	$5 + 0.5$	0.2	76.85	2 887 284	76.85	329	0
80	$5 + 1$	0.1	77.315	4 431 462	77.315	269	0
80	$5 + 1$	0.2	77.294	10 166 557	77.294	269	0
90	$5 + 0.5$	0.1	N/A	3 293 544	85.506	1 414	–
90	$5 + 0.5$	0.2	N/A	4 737 207	85.343	1 614	–
90	$5 + 1$	0.1	N/A	8 276 707	86.762	2 170	–
90	$5 + 1$	0.2	N/A	8 956 056	86.75	1 502	–

Table 1. Results of algorithms based of CPLEX and PSO solvers

As it can be seen from Table 1, CPLEX is not able to solve problems of dimension 90 with given parameters. Similarly, CPLEX cannot solve problems of greater dimensions and CPLEX throws out an out of memory exception. On the other hand, developed PSO method in all tests reached optimal values very fast.

In the second test, the developed PSO method is executed on instances with dimensions up to 900 locations with parameters: number of facilities $P = 20, 25, 30$, radius $R = 2, 2.5, 3$ and fuzzy variables $f_r = 0.5, 1$, $f_t = 0.1, 0.2$. Table 2 shows results obtained on instances with dimensions $n = 100$, $n = 500$, $n = 800$ and $n = 900$ with 20, 25 and 30 facilities, radius of coverage $2 + 0.5, 2.5 + 0.5$ and $3 + 0.5$ and fuzzy distances 0.1 and 0.2. All computational results are given on `http://www.matf.bg.ac.rs/~maricm/fmclp`. Computational results contain the result, time of the best solution, total execution time, average gap ($agap$) and standard deviation (σ). The average gap ($agap$) is calculated as $agap = \frac{1}{N}\sum_{i=0}^{N} gap_i$, where N represents the number of PSO runs on the same instance (in this study $N = 10$). Symbol gap_i represents the gap of PSO's results obtained in the i^{th} run. In all runs, gap_i is evaluated with respect to best-known solution ($Best.Sol$) with formula $gap_i = 100 \cdot \frac{sol_i - Best.Sol}{Best.Sol}$. As it is well known, optimality for results in tests cannot be proven and thus the solution obtained by the algorithm is taken for the best PSO solution. Standard deviation is calculated by the known formula $\sigma = \sqrt{\frac{1}{N}\sum_{i=0}^{N}(gap_i - agap)^2}$.

Algorithm is tested 324 times on different input data. In 54.94 % percent of instances, algorithms have $agap$ below 2 %. Average obtained gap in all tests is 1.86 and average standard deviation is 1.15. Maximal gap 5.87 and maximal standard deviation 2.43 are obtained on instance with parameters $n = 800$, $P = 30$, $R + f_R = 2 + 0.5$ and $f_t = 0.1$.

As we mentioned before, CPLEX is not able to find solutions for instances with more that 90 nodes. On other hand, Tables 1 and 2 show that PSO is capable to solve all tested instances in acceptable time and it reaches all CPLEX results, so it is reasonable to conclude that the developed method is competitive for solving the proposed model. Tests confirmed a known fact that any increase in a fuzzy value in the radius of coverage always gives a solution with more covered locations. On the other hand, changing fuzzy values in distance variables gives different results in different instances – sometimes it covers more locations, but sometimes it covers less locations. Reason for this property is the proposed method for calculation of the degree of coverage and intersection of triangular and right shoulder fuzzy numbers illustrated in Figure 3.

6 CONCLUSION AND FUTURE WORK

In this paper a new model of FMCLP has been proposed and fuzzy numbers are used for modeling uncertainties. Particle swarm optimization metaheuristic has been proposed for solving of this model and testing results have been presented.

Further work related to this paper will be the implementation of other metaheuristics and hybrid methods for solving the proposed model and comparing their efficiency. The introduction of similar fuzzy conditions in other combinatorial optimization problems will be a next direction in the future research. The authors of this paper are currently working on creating fuzzy model for minimal covering location problem.

n	P	$R+f_r$	f_l	*Best.Sol*	*Sol.Time*	*Total.Time*	*agap* %	σ
100	20	$2+0.5$	0.1	67.871	1 557	7 871	0.34	0.50
100	20	$2+0.5$	0.2	67.362	2 143	8 052	0.22	0.25
100	20	$2.5+0.5$	0.1	80.755	1 794	7 525	0.80	0.68
100	20	$2.5+0.5$	0.2	80.828	1 670	7 434	0.26	0.38
100	20	$3+0.5$	0.1	90.353	4 220	9 897	0.88	0.61
100	20	$3+0.5$	0.2	90.153	3 390	9 102	0.24	0.24
100	25	$2+0.5$	0.1	76.326	3 521	9 328	0.40	0.41
100	25	$2+0.5$	0.2	75.846	4 117	10 004	0.22	0.15
100	25	$2.5+0.5$	0.1	86.502	3 995	9 635	0.15	0.22
100	25	$2.5+0.5$	0.2	86.549	5 043	10 761	0.46	0.36
100	25	$3+0.5$	0.1	95.679	2 456	8 049	0.33	0.39
100	25	$3+0.5$	0.2	95.574	4 698	1 0367	0.76	0.84
100	30	$2+0.5$	0.1	82.735	11 353	17 254	0.61	0.49
100	30	$2+0.5$	0.2	82.124	3 377	9 351	0.50	0.37
100	30	$2.5+0.5$	0.1	91.442	4 120	9 952	0.78	0.48
100	30	$2.5+0.5$	0.2	91.466	10 481	16 234	0.33	0.46
100	30	$3+0.5$	0.1	99.211	2 514	8 076	0.25	0.18
100	30	$3+0.5$	0.2	99.458	4 305	10 040	0.68	0.67
500	20	$2+0.5$	0.1	243.011	260 548	391 713	2.86	1.83
500	20	$2+0.5$	0.2	239.779	524 341	654 685	0.94	0.92
500	20	$2.5+0.5$	0.1	308.655	224 556	350 158	1.35	0.92
500	20	$2.5+0.5$	0.2	313.010	329 688	455 848	3.02	1.91
500	20	$3+0.5$	0.1	375.680	170 115	291 896	4.09	1.82
500	20	$3+0.5$	0.2	369.044	359 620	483 657	2.40	1.94
500	25	$2+0.5$	0.1	279.360	378 229	505 969	2.18	1.24
500	25	$2+0.5$	0.2	289.144	458 709	587 979	4.20	1.88
500	25	$2.5+0.5$	0.1	359.092	363 496	488 632	2.60	1.38
500	25	$2.5+0.5$	0.2	356.848	241 337	368 029	3.26	1.51
500	25	$3+0.5$	0.1	413.674	322 573	446 136	1.60	1.61
500	25	$3+0.5$	0.2	415.562	376 987	499 340	2.29	1.14
500	30	$2+0.5$	0.1	312.867	407 482	535 665	1.00	0.71
500	30	$2+0.5$	0.2	315.917	540 255	667 017	2.12	1.11
500	30	$2.5+0.5$	0.1	394.563	660 241	782 923	2.31	1.45
500	30	$2.5+0.5$	0.2	388.431	290 587	415 886	2.40	1.51
500	30	$3+0.5$	0.1	443.940	97 468	216 466	1.25	1.11
500	30	$3+0.5$	0.2	455.415	678 139	791 845	2.32	1.50

n	P	$R + f_r$	f_l	*Best.Sol*	*Sol.Time*	*Total.Time*	*agap* %	σ
800	20	$2 + 0.5$	0.1	355.118	699 611	1 046 569	4.30	1.84
800	20	$2 + 0.5$	0.2	343.949	773 755	1 127 638	2.39	1.29
800	20	$2.5 + 0.5$	0.1	463.026	1 199 970	153 7446	4.53	2.12
800	20	$2.5 + 0.5$	0.2	454.472	834 303	1 173 370	2.93	1.85
800	20	$3 + 0.5$	0.1	574.867	873 483	1 196 154	5.85	2.41
800	20	$3 + 0.5$	0.2	549.791	736 213	1 058 464	1.32	0.67
800	25	$2 + 0.5$	0.1	405.771	633 905	976 338	2.09	1.61
800	25	$2 + 0.5$	0.2	408.241	694 939	1 034 444	1.98	1.99
800	25	$2.5 + 0.5$	0.1	529.771	772 146	110 4117	3.28	1.48
800	25	$2.5 + 0.5$	0.2	523.437	609 647	937 569	3.04	1.57
800	25	$3 + 0.5$	0.1	623.804	279 271	600 143	1.67	1.12
800	25	$3 + 0.5$	0.2	638.047	809 213	1 126 970	3.72	1.75
800	30	$2 + 0.5$	0.1	478.642	1 452 437	1 786 968	5.87	2.43
800	30	$2 + 0.5$	0.2	460.678	757 960	1 099 268	2.65	1.52
800	30	$2.5 + 0.5$	0.1	584.299	464 765	786 487	2.36	1.28
800	30	$2.5 + 0.5$	0.2	577.633	1 086 267	1 412 218	2.56	1.52
800	30	$3 + 0.5$	0.1	679.925	901 983	1 223 607	1.37	0.95
800	30	$3 + 0.5$	0.2	672.031	271 563	580 390	1.11	0.93
900	20	$2 + 0.5$	0.1	396.458	746 652	1 192 244	4.83	1.90
900	20	$2 + 0.5$	0.2	382.122	516 080	971 273	2.54	1.61
900	20	$2.5 + 0.5$	0.1	506.792	1 223 832	1 656 887	1.84	1.12
900	20	$2.5 + 0.5$	0.2	504.608	890 448	1 324 987	2.33	1.51
900	20	$3 + 0.5$	0.1	607.633	1 399 932	1 825 608	1.71	1.08
900	20	$3 + 0.5$	0.2	618.172	195 804	615 621	2.60	1.05
900	25	$2 + 0.5$	0.1	452.046	678 070	1 119 510	2.28	1.35
900	25	$2 + 0.5$	0.2	451.013	587 142	1 034 476	1.52	1.49
900	25	$2.5 + 0.5$	0.1	583.544	410 512	834 589	2.54	1.47
900	25	$2.5 + 0.5$	0.2	577.736	995 791	1 432 477	1.89	1.09
900	25	$3 + 0.5$	0.1	722.648	344 676	760 136	4.63	1.95
900	25	$3 + 0.5$	0.2	695.761	1 037 067	1 457 792	2.34	1.65
900	30	$2 + 0.5$	0.1	518.665	608 909	1 060 774	2.50	1.78
900	30	$2 + 0.5$	0.2	507.704	148 065	582 565	2.16	1.60
900	30	$2.5 + 0.5$	0.1	647.080	361 156	779 588	2.20	0.99
900	30	$2.5 + 0.5$	0.2	640.107	407 976	831 076	0.71	0.53
900	30	$3 + 0.5$	0.1	768.443	862 697	1 262 284	2.36	1.15
900	30	$3 + 0.5$	0.2	766.596	1 263 250	1 660 216	2.68	1.02

Table 2. Results of PSO method on instances with dimension $n = 100$ and $n = 900$

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