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ACCURATE ANALYSIS OF THE SPATIAL PATTERN OF REFLECTED LIGHT AND SURFACE ORIENTATIONS BASED ON COLOR ILLUMINATION

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Abstract. 3D recovery approaches require a variety of clues to obtain shape information. The shape from shading (SFS) method uses shading variations in images to estimate depth maps. Although shading contains detailed information, it causes some well-known ambiguities such as convex-concave ambiguity. In this study, a system installation using red, green, and blue illumination and an algorithm processing reflections on the surface were proposed to accurately analyze surface orientations and solve ambiguity problems. The algorithm evaluated combinations of light hitting the surface from different directions and detailed surface orientations to avoid erroneous predictions. The proposed system was tested with eight different methods in the literature developed from the earliest times to the present, and the initially erroneously predicted surface orientations were improved. Consequently, the correct orientation of the surface points was determined by removing the ambiguities in images taken without considering the location of illumination, and all the tested methods provided successful results using the proposed system.

Keywords: 3D reconstruction, shape from shading, scene understanding, ambiguity

Mathematics Subject Classification 2010: 68-04

1 INTRODUCTION

Obtaining 3D shapes of objects in the images is one of the main and interesting research topics in computer vision. 3D reconstruction approaches try to obtain depth information from various clues provided by the images. The spatial pattern of light reflected from surfaces, the shading, is one of the most important clues providing detailed information about 3D shapes [1, 2, 3]. Shape from shading (SFS) was initially introduced by Horn [4] as the solution of the first-order nonlinear partial differential equations. It is a classical 2D to 3D inverse problem in computer vision [5]. SFS tries to obtain 3D shape information by using shading as a clue to understand how the observed intensity variation across surfaces of objects provides conclusions about the local topography. It is required to make various assumptions in order to find a solution. In literature, SFS methods have been classified in many different ways, and representative methods selected from such classifications have been compared with each other [6, 7].

In addition to the popularity of mathematical and algorithmic developments about SFS methods, the shape information obtained with SFS is used in many different fields such as surface topography and terrain analysis [8, 9], biometric studies [10, 11], industrial quality control [12, 13], and medical diagnosis and treatment [14, 15]. Although shading serves as an important means for shape reconstruction, it is an ambiguous clue of relief [3, 16]. SFS is one of the powerful 3D reconstruction methods, but it spans some well-known ambiguities, and is considered ill-posed due to the ambiguities such as convex-concave ambiguity, and bas-relief ambiguity [5, 17, 18]. The convex-concave ambiguity is of interest not only in computer vision, but also in the context of understanding models of human perception [19].

Pentland [20] implied that the convexity of the surface cannot be determined unless something is known about the direction of illuminant. Among the works that considered the question of uniqueness and ambiguity, most of them considered restricted versions of the problem such as constraints and limitations on the environment (surface, the light source, camera model, multi-view images), or information used in the solution such as boundary conditions, singular points, or graphs [17, 21]. The ambiguity in Figure 1 is due to a change in the estimation of illumination parameters.

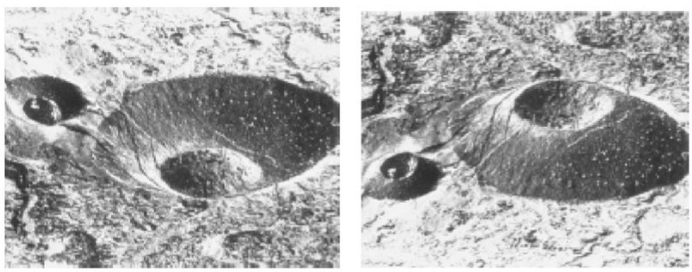


Figure 1. SFS ambiguity: perceiving two craters or volcanoes according to the position of light source [5, 20, 22]

The brightness variations, observed in a single image, indicate variations in the normal, and in geometry [23]. In a single gray level image, both the indentation (yellow-green gradients in Figure 2), and the protrusion (red-blue gradients in Figure 2) are possible geometric configurations. Most shape from shading methods require a highly controlled illumination, and often fail when deployed in real-world conditions [23].

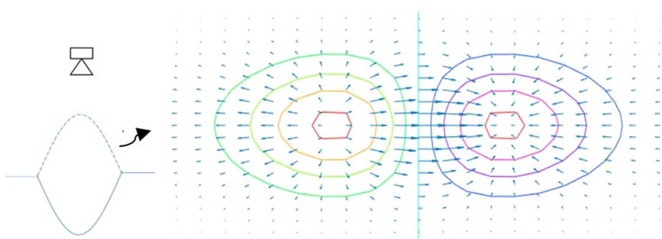


Figure 2. Possible geometric configurations of convex-concave ambiguity [24]

The main difference here is to suggest a method that will allow the algorithms to perform an accurate surface analysis by two images in total using color reflections and shading. Unlike photometric stereo setups, color reflections give information about surface orientations, and the effect of the light source's position is reduced by them even on surfaces with near-point illumination, thus allowing the existing SFS algorithms to create satisfying surface reconstructions. The wall in Figure 3 is illuminated only with red, green, and blue colors. When illuminated simultaneously with all three colors, the wall surface is perceived as white, but when there is an obstacle that refracts the light, only the reflections of certain colors are perceived.

Figure 3. A wall illuminated by red, green, and blue color illuminants. Different color patterns form on the wall as the result of the combination of lights.

In this paper, a calibrated illumination environment, specifically chosen in order to be directly informative about shape, was used to improve the surface reconstruction of well-known SFS methods. The surface was illuminated by three (red, green, blue) monochromatic directional light sources lit from different directions. The images of the synthetic surfaces, produced in the modeling environment using a point light source, were initially assigned to eight basic SFS methods without information, and then assigned with color reflection information, and the surfaces created by the methods were examined. Thanks to the developed algorithm interpreting color patterns on the surface, efficient and high-quality surface recovery from a single image is possible. The proposed framework is shown in Figure 4.

Basically, the color distributions, formed on the surface through illumination at the same time by different color light sources, were processed algorithmically, and details about the orientation of the surface points were provided for the SFS methods. Concave and convex regions were detected without being affected by the position of the light source, and a system was proposed for accurate analysis of surface orientations. The algorithm was tested on two of the preferred synthetic surfaces in the ambiguity problems [18, 21], and the deviations and errors of the SFS methods arising from the change of the light source's position were eliminated. By

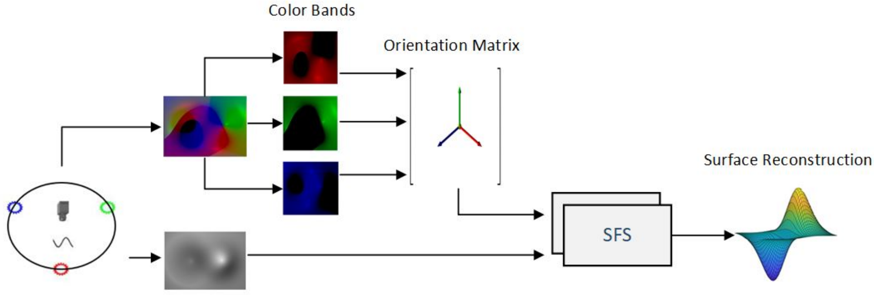


Figure 4. Proposed framework

being aware of the locations of illuminants, and by calculating the color reflections, surface normals and orientations were determined correctly.

The rest of the study can be summarized as follows: In Section 2, literature studies are discussed. In Section 3, the modeling of the proposed system is described. In Section 4, the surface reconstructions and comparisons are given in detail. Finally, future studies and results are discussed.

2 LITERATURE REVIEW

The traditional shape from shading, with a single light source and Lambertian reflectance, is a challenging problem. The constraints implied by the illumination are not sufficient to specify the local orientations [17]. In order to solve the general ambiguities, many approaches have been emphasized in the literature.

Woodham [25] introduced a technique called photometric stereo. The ambiguities in determining the local surface orientation from intensity measurements were eliminated by varying the direction of illumination between successive images. Johnson and Adelson [17] estimated surface normals from a single image of a diffuse object under natural illumination. They assumed uncontrolled natural illumination variability to reduce ambiguity in surface orientation. The surface normals of a diffuse object were estimated from a single image captured under known but uncontrolled illumination. Xiong et al. [26] developed a framework for obtaining shape information from diffuse shading, and used the combination of dense local estimation and globalization for traditional SFS assuming known albedo and a single known directional light. Chakrabarti and Sunkavalli [27] employed a similar framework developed in [26], and obtained geometric details from images captured under the RGB-photometric stereo setup in the presence of spatially varying albedo. A diffused surface was illuminated by three-directional monochromatic light sources, similar to the environment described here. They performed local inference on a dense set of overlapping patches to produce distributions of candidate shapes for every patch and each candidate corresponded to a different assumed albedo. Harmonizing the local distributions resulted in the creation of a surface normal map. Prados and

Faugeras [5] described basic ambiguities of SFS, and assumed that all the parameters of the light source, the surface reflectance, and the camera were known. Yu et al. [28] used Microsoft Kinect to resolve ambiguities in SFS, and proposed a shading-based shape refinement algorithm. Noisy, incomplete depth maps obtained were used to resolve bas-relief ambiguity, and they clustered the pixels with similar normal directions. Zhu and Shi [29] used the singular points, the points where surface normals are frontal to the illumination direction, in order to determine the local ambiguity. Abada et al. [21] stated that singular points are necessary but not sufficient to resolve local ambiguity, and that it is required to have further information. They also stated that whenever the number of edges between the singulars points increases, the ambiguity decreases. Figure 5 is taken from their study to show local ambiguity for a graph configuration, and the surface shown was also studied in this study.

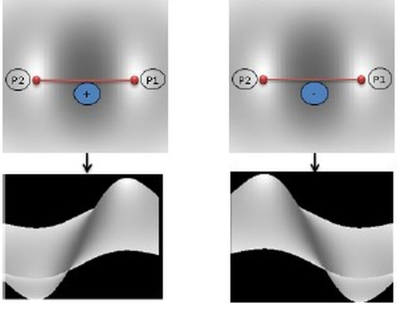


Figure 5. Local ambiguity for a graph configuration [21]

Unlike single-view reconstruction and a utilized orientations matrix derived from color illuminated surface, Quéau et al. [23] introduced a variational method for multi-view SFS under natural illumination. A fusion of multiple-view reconstruction and SFS was proposed for accurate dense reconstructions. The key idea is to couple partial differantial equation based solutions for single-image based SFS problems across multiple images and multiple color channels by means of a variational formulation. They modeled the brightness variations of each color channel and each image through a partial differential equation, and used an Alternating Direction Method of Multipliers (ADMM) algorithm to solve the nonlinearly coupled optimization problem. Belhumeur et al. [16] emphasized that neither shading nor shadowing reflects the true 3D structure of the object from a single viewpoint. Henderson and Ferrari [30] illustrated that directional color illumination provides strong clues for surface orientation. Class-specific 3D reconstruction from a single image, and the generation of new 3D shapes were studied. Their method exploited shading in training images, and tested in both illuminations by three directional color illuminants, and by illumination with one white directional illuminant. Breuss et al. [19] explained the convex-concave ambiguity in the perspective SFS model. By addressing the light source attenuation factor, they looked for an answer to the question of whether all ambiguities are being eliminated by the use of the perspective SFS model, or not. They showed that ambiguity situations are still occurring in practical calculations, and proposed an algorithm for complex surfaces. 3D recovery methods are frequently used in inspection works [12, 13, 31], and the method proposed here covers especially inspectable surfaces and systems in which color illumination can be installed.

3 SYSTEM PROPOSITION

A system based on three color illumination was proposed to prevent incorrect 3D reconstruction due to ambiguities and shading tricks. For example, the surface, preferred in ambiguity studies [18, 21], given in Figure 6, can be considered.

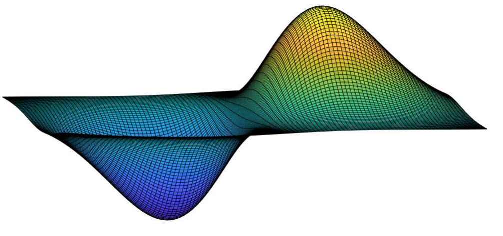


Figure 6. Example of synthetically formed surface with indentation and protrusion regions

The above surface was obtained using the formulation in Equation (1):

$$Z = 2 * \left(\frac{1}{e^{((x-5)^2+y^2)}} - \frac{1}{e^{((x+5)^2+y^2)}} \right) \{x, y | x, y \in R\}. \tag{1}$$

By changing the location of the light source of the above surface, many different shading images can be obtained. Assuming that the observer's direction (camera view) is fixed, some image examples and (x, y, z) locations are shown in Figure 7.

Satisfactory convergent results can be obtained if the shading information is used to obtain the above surface shape in cases where the indentations and protrusions of the surface are compatible with the shading image and the illumination direction. Sample gray level image by the use of the distant light source, and the shape estimation of the surface in Figure 6 are given in Figure 8.

The surface has been properly estimated by considering the low surface points as darker, and the high surface points as brighter. When the surface in Figure 6 is illuminated from different positions, different shading images and surface orientations may cause erroneous estimation. In Figure 9, the situation, where the same surface is illuminated from above by a near-point light source without changing the observer direction, is discussed. In such a case, the gray level image, and the surface prediction of the most shading interpreter algorithms will be erroneous.

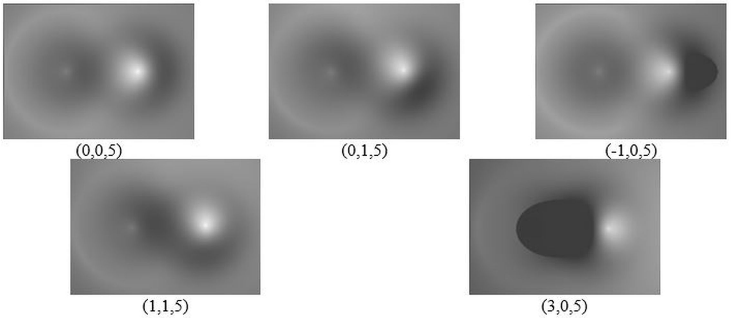
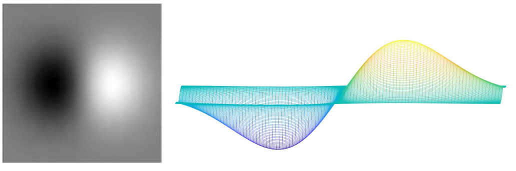


Figure 7. Images of the same surface illuminated from different locations

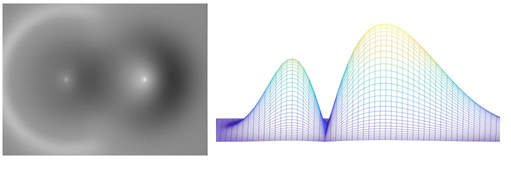


a) b)

Figure 8. a) The gray level image of the surface in Figure 5, and b) predicted surface shape with high accuracy by interpreting shading information

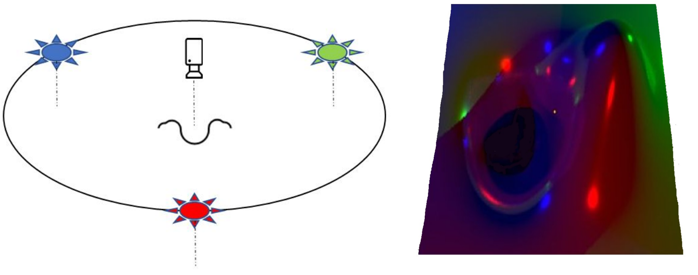
By illuminating the surface at the same time with different color light sources, it is intended to process the color distributions on the surface algorithmically, and to provide accurate information about the orientation of the surface points to SFS methods. The sample surface, and scene design were addressed using the Blender 3D [32] modeling environment in order to examine the color reflections. Color illuminants are positioned on a certain small portion of the surface. The scene design, and sample color reflections (rendering) formed on the surface are shown in Figure 10.

The images obtained by illuminating the surface from three different directions using red, green, and blue colors simultaneously and separately are shown in Fig-



a) b)

Figure 9. a) The gray level image of the surface in Figure 6 illuminated by near-point light source, and b) approximate surface estimation according to the some shading-based algorithms



a) b)

Figure 10. a) Proposed scene design and b) sample rendering of the given surface

ure 11. The developed method sorts the input color image as per all color bands, and creates color distribution masks.

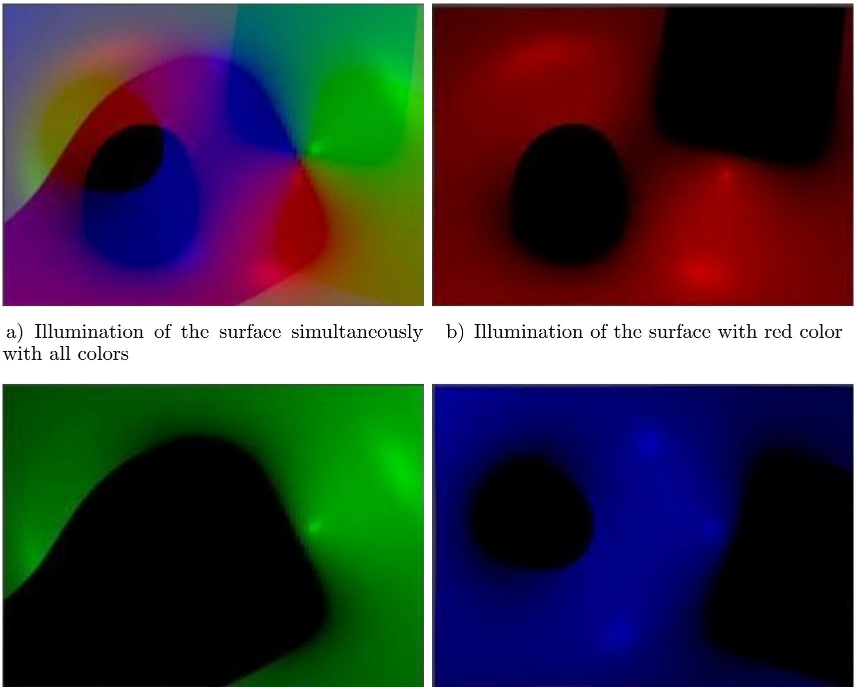
The color decomposition is achieved by converting each color component in the received RGB image (ImRGB) into separate 2D color band arrays according to the image color map. Separate color bands and mask images are obtained by applying the threshold value $ImRGB_{[R,G,B]} > 0$ to the generated sequences. Thanks to the generated 2D arrays, the mixed color bands can easily be detected by comparing specific color bands. The calculated band images of the color reflections are shown in Figure 12. The regions, where none of the red, green, and blue colors are lit, are the black band, and the regions lit by the red and green colors are yellow band, and the regions lit by the red and blue colors are magenta band, and the regions lit by the green and blue colors are cyan band, and the regions lit by all three colors together are masked as a white band.

A separate histogram can also be used to detail red, green, and blue pixels as well as total pixel counts. Figure 13 illustrates the histogram of all colors between [80–120] values when all color band arrays are scaled to a gray level of [0–255]. The threshold values can be updated and objects or noise under certain pixels in the image eliminated by using specific threshold values or methods such as Otsu.

Even if different shading images are obtained by illuminating the surface from different positions, thanks to the correct processing of color reflections, SFS algorithms can distinguish concave and convex regions correctly, and can make correct interpretations about surface orientations. After processing the decomposed color bands, the erroneous surface orientation shown in Figure 9 b) was calculated more accurately as in Figure 14 b).

4 DETERMINATION OF ORIENTATIONS AND TEST RESULTS

In order to examine the shading sensitivity of SFS methods, and the effectiveness of the proposed approach on different methods, synthetically produced surfaces con-



a) Illumination of the surface simultaneously with all colors

b) Illumination of the surface with red color

c) Illumination of the surface with green color

d) Illumination of the surface with blue color

Figure 11. Illumination of the surface with different colors

taining single indentation were used. The gray level image of the surface was first assigned to eight different SFS methods, and 3D surface estimates were obtained. Then, the same image was assigned to the same methods by using the developed surface orientation algorithm, and the new results obtained were compared with the previous surface estimates.

4.1 Description of Selected Methods

Lee and Rosenfeld [33] assumed an isotropic distribution of surface orientation. With the help of a coordinate system with one axis in the assumed direction of the light source, a method for estimating Lambertian surface shape from shading information was developed. The image was rotated from viewer to light source coordinates to compute the intensity gradient in terms of light source coordinates.

Frankot and Chellappa [34] described a method for enforcing integrability, projecting the possibly nonintegrable surface slope estimates onto the nearest integrable surface slopes. The surface slopes were represented by finite sets of

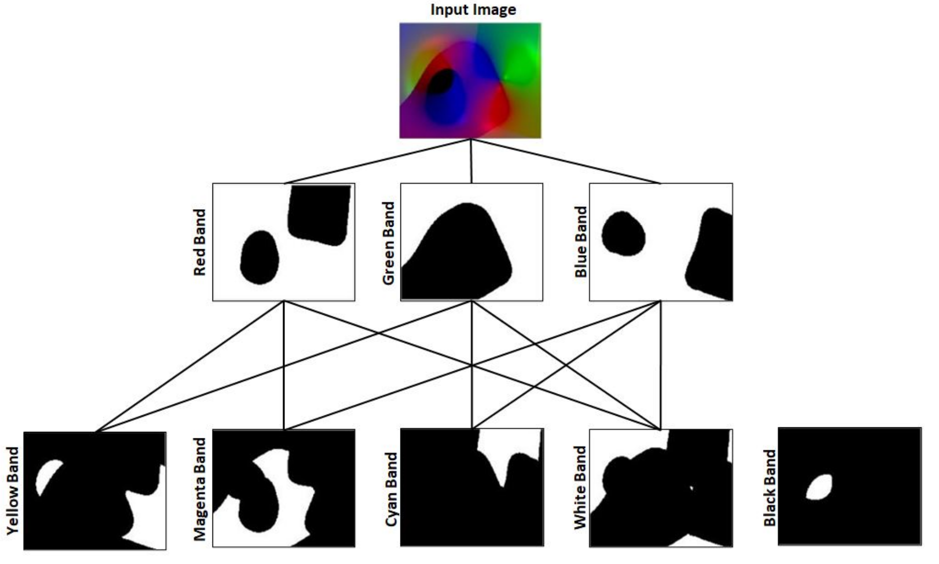


Figure 12. Decomposition of the surface illuminated with red, green and blue colors into color bands

orthogonal integrable basis functions. The integrability constraint led to the reconstruction of surface height by integrating surface slope estimates.

Pentland [35] employed the linear approximation of the reflectance function in terms of the surface gradient. To get a closed form solution for the depth at each point, Fourier transform was used on the linear function.

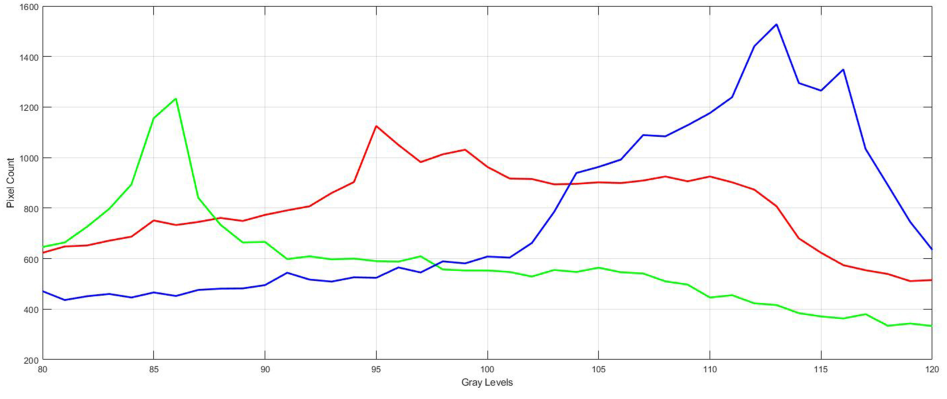
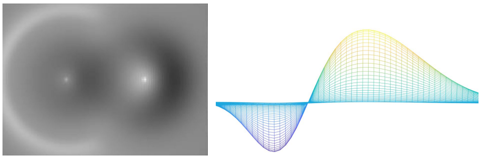


Figure 13. The histogram of all R, G, B bands in the value range [80–120]



a) b)

Figure 14. Surface estimation after processing color information

Tsai and Shah [**36**] utilized the discrete approximations of gradients using finite differences in order to linearize the reflectance map in terms of depth (Z). Using a Jacobi iterative scheme, their algorithm recovered the depth at each point.

Zheng and Chellappa [**37**] computed the surface by implementing the smoothness constraint requiring the gradients of reconstructed density to be close to the gradients of the input image. Euler equations were simplified by taking the first-order Taylor series of the reflectance map.

Barron and Malik [**38**] introduced a model, SIRFS (shape, illumination, and reflectance from shading), taking a single image and producing the estimate of the shape, surface normals, reflectance, shading, and illumination. They defined the approach as an optimization problem recovering surface characteristics under specific priors.[1]

Quéau et al. [**39**] presented a numerical solution based on an augmented Lagrangian approach for solving a generic PDE-based SFS model which handles a variety of models for the camera and the lighting. They offered an ADMM approach to solve the resulting system of PDEs, which separates the difficulty due to non-linearity from that related to gradient dependency.

Kotan et al. [**40**] obtained the depth map by using different spatial coefficients of numerical gradients of images as an initial state and linearizing the reflectance map in terms of depth. A hybrid linearization based SFS is presented.[2]

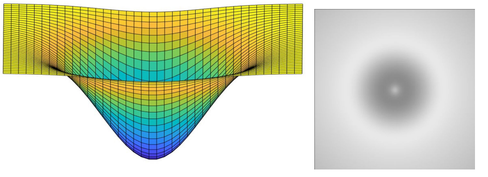
4.2 Test Results

The surface shown in Figure 15 a) was generated by using Equation (2) and the image in Figure 15 b) was obtained as the result of illuminating the surface by using a near point light source.

$$Z = \frac{-2}{e^{((x+0.1)^2+y^2))}}\{x, y|x, y \in R\}. \tag{2}$$

[1] Sample code implementations are available at `https://jonbarron.info/`.

[2] Sample code implementations are available at `https://github.com/muhammedkotan/hybridSFS`.



a) b)

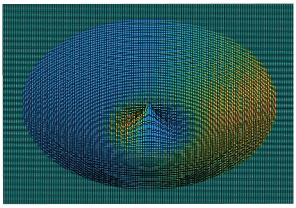
Figure 15. a) Synthetically generated surface with an indentation and b) the result of illuminating from above with a near-point light source

Many SFS algorithms assume that although the direction of illumination is provided, the surface begins to rise as the brightness increases at the deepest point of the surface. Prior to provision of the color reflection information, the surfaces produced by the eight methods for the image in Figure 15 b) are shown in Figure 16.

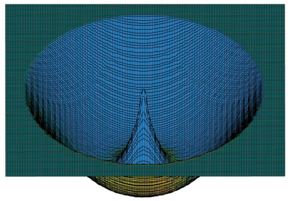
When the results are visually evaluated, it is obvious that the methods address the brightness in the center of the indentation region as an increase, and produce inaccurate surface estimations. The color image of the surface in Figure 15 a), and decomposed color bands in the application developed are shown in Figure 17.

As the selected methods perceive the gradations at the center of the indentation area as elevation, the incorrectly reconstructed surface shapes are improved by using the color reflection algorithm developed. Since the locations of color light sources are known, the orientation of the surface points can be determined easily by interpreting the color bands. Color reflections are analyzed, and the surface is decomposed into color bands. The direction of the surface points is determined using color patterns and illumination directions. In addition to the visual comparison of the orientation, we compared the depths produced by the methods with the ground-truth depth map as benefiting from various metrics used in image analysis: Mean Squared Error (MSE), Structural Similarity Index (SSIM), Peak Signal-to-Noise Ratio (PSNR). Regarding the first metric, the lower the value is, the more convergent the result is. The other two metrics show the similarity ratio, and the higher value indicates a more similar result. The new surface estimates, created by the methods for the same surface using the color processing algorithm, are shown in Figure 18.

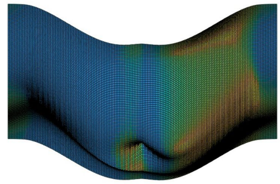
Note that the algorithms contain many parameters and pre-processing steps. In addition, some of them have an iterative nature. Therefore, they can produce different results according to the selected parameters and values. The emphasis here is not on the performance comparisons of the algorithms against each other but on the proposed method based on color reflection analysis, it is showing that the results are better visually and metrically. The technique developed improves the performance of all SFS methods in all of the selected metrics and provides 100 % performance.



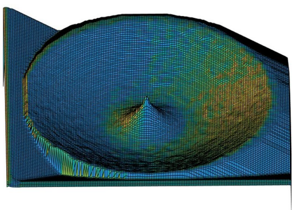
a) Lee and Rosenfeld



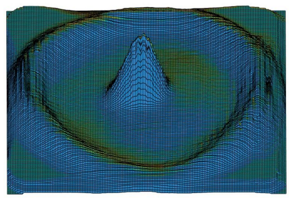
b) Frankot and Chellappa



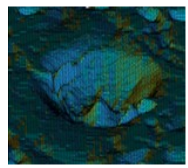
c) Pentland



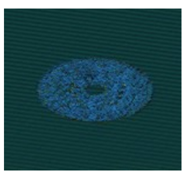
d) Tsai and Shah



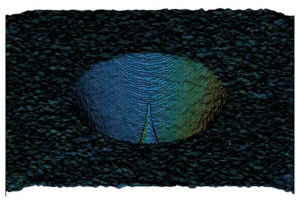
e) Zheng and Chellappa



f) Barron and Malik



g) Quéau et al.



h) Kotan et al.

Figure 16. The surfaces estimated by eight different SFS methods

SFS Method	Before-Proposed System			After-Proposed System		
	MSE	*PSNR*	*SSIM*	*MSE*	*PSNR*	*SSIM*
Lee and Rosenfield	0.0619	12.0796	0.8496	**0.0274**	**15.6299**	**0.8937**
Frankot and Chellappa	0.0219	16.6041	0.8831	**0.0201**	**16.9723**	**0.9340**
Pentland	0.0214	16.6931	0.9163	**0.0162**	**17.9112**	**0.9377**
Tsai and Shah	0.1622	7.8999	0.6194	**0.1203**	**9.1969**	**0.6644**
Zhang and Chellappa	0.6699	1.7402	0.1000	**0.0114**	**19.4290**	**0.9206**
Barron and Malik	0.0041	19.0570	0.6793	**0.0033**	**19.9514**	**0.8181**
Quéau et al.	0.3246	4.8862	0.2903	**0.2171**	**6.6329**	**0.4084**
Kotan et al.	0.0515	12.8801	0.8529	**0.0488**	**13.1183**	**0.8569**

Table 1. Performance measurement of the proposed system

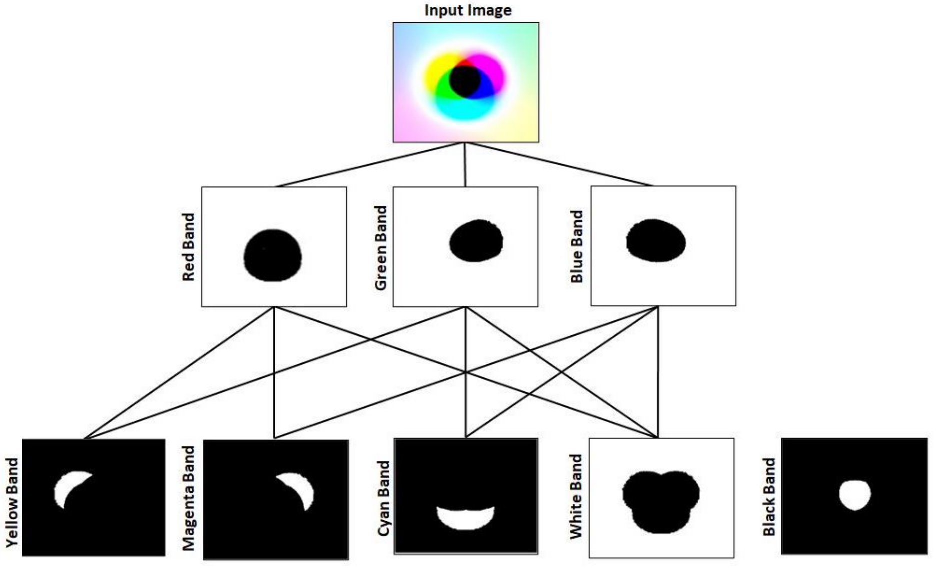
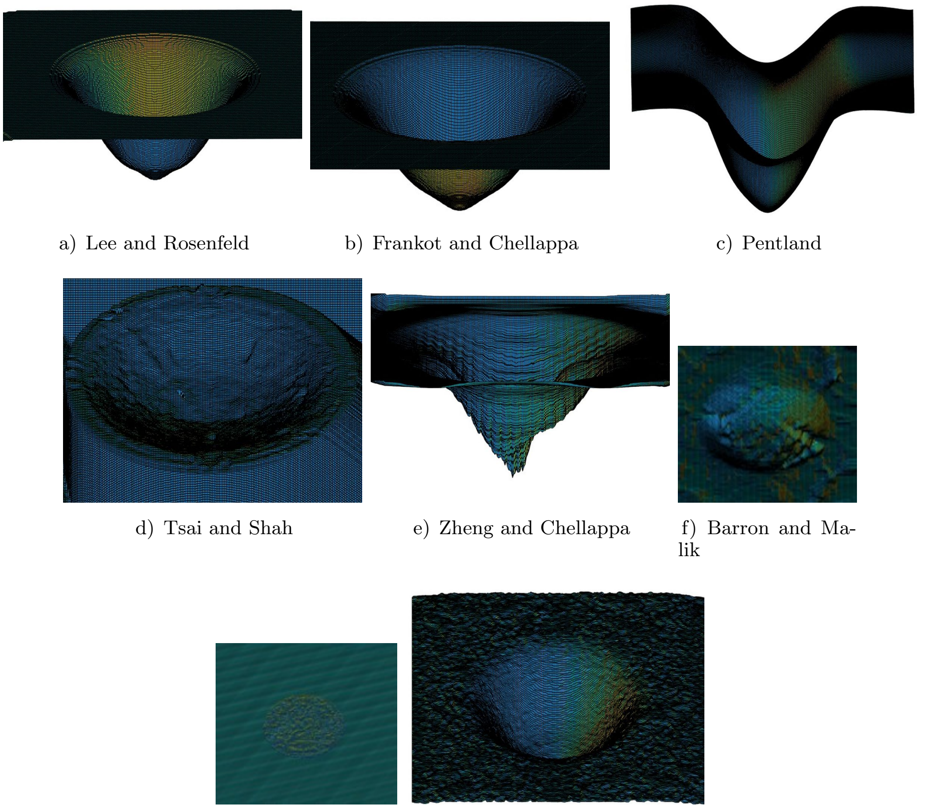


Figure 17. Decomposition of the surface illuminated with red, green and blue colors into color bands

5 FUTURE WORK AND CONCLUSION

Using a system prepared in a 3D modeling environment, a solution was proposed for the ambiguity of surface reconstructions, and especially for the convex–concave ambiguity. The gray level image, and color reflections of the surface illuminated with 3 different colors at a certain height and position were interpreted correctly, and the orientations of the surface regions were predicted more accurately regardless of the lighting direction. Synthetic, white, and matte surfaces, produced in a 3D modeling environment, were used. The scene was illuminated simultaneously with red, green, and blue color illuminants. The orientations of the surface points were correctly analyzed by using the color patterns formed on the surface. The image, taken from the above of the surface, was assigned to eight different well-known SFS methods as the input image, and they ensured the prediction of the surface shape. The selected SFS methods were first applied on the generated synthetic surfaces without the knowledge of color reflections. And then they were applied to the same surfaces with the color reflection algorithm developed. Due to the illumination position and shading, the initially estimated incorrect surface reconstructions were improved by the use of the proposed method, and the SFS methods were enabled to make a more accurate surface estimation as independent from the illumination direction. The results were compared qualitatively and quantitatively. The technique developed enabled the methods to calculate surface orientations accurately, and to estimate more convergent surface reconstructions.



a) Lee and Rosenfeld b) Frankot and Chellappa c) Pentland

d) Tsai and Shah e) Zheng and Chellappa f) Barron and Malik

g) Quéau et al. h) Kotan et al.

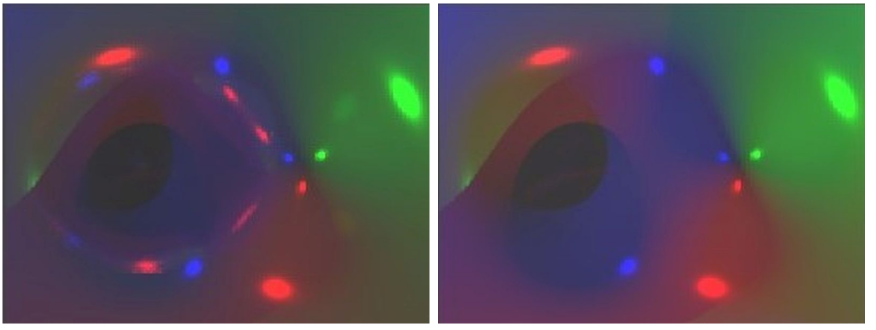
Figure 18. The new surfaces estimated by eight different SFS methods

The application spans some limitations, and they can be addressed in future studies:

1. The surfaces used were based on the assumption of matt and white-colored surfaces with very low reflectivity. Complex images may occur due to the reflection of colors in highly reflective surfaces such as metals. Sample images, showing different reflections that may occur on the surface when the same surface has parametrically different reflectivity coefficients, are shown in Figure 19.

2. Color bands, reflected by the surface, should be considered on different color surfaces. The algorithm developed may need to be customized according to the object's surface.

3. The studies were carried out theoretically using virtual environments. The reaction of the system under real environmental conditions can be investigated in the next stage of the study.



a) b)

Figure 19. Different reflection images obtained by changing the surface reflectivity coefficient

In future studies, highly reflective surfaces, or improvements and customizations of the algorithm can be studied according to the surface to be examined. It can be ensured for the algorithm developed to be used for parts and surfaces having different properties and sizes. With the adaptation of the system to the real environment, the proposed design can be used in many fields such as inspection and surface analysis systems. The algorithm developed can be used to detect surface orientations or defective areas by eliminating reflections, and by using appropriate hardware components. In industrial quality control systems, it can be used to detect defects such as dents, scratches, holes, cracks, etc.

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