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A NOTE ON THE MDCT/MDST AND PSEUDOINVERSE MATRIX

Vladimír Britaňák

Institute of Informatics
Slovak Academy of Sciences
Dúbravská cesta 9
845 07 Bratislava, Slovak republic
e-mail: vladimir.britanak@savba.sk

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Abstract. The modified discrete cosine transform (MDCT) and modified discrete sine transform (MDST) both for the evenly and oddly stacked systems are perfect reconstruction cosine/sine–modulated filter banks based on time domain aliasing cancellation (TDAC) employed in the current international audio coding standards and commercial audio compression products. Based on the matrix representation of MDCTs and MDSTs it is shown that the transposed MDCT and MDST matrices are actually the pseudoinverses of their corresponding forward transform matrices. The pseudoinverse matrix and its properties provide an elegant mathematical tool to characterize the MDCT/MDST as the analysis/synthesis filter banks in the matrix representation.

Keywords: Modified discrete cosine transform, modified discrete sine transform, modulated lapped transform, modulated complex lapped transform, pseudoinverse matrix

1 INTRODUCTION

The modified discrete cosine transform (MDCT) and modified discrete sine transform (MDST) are perfect reconstruction cosine/sine-modulated filter banks based on time domain aliasing cancellation (TDAC) employed in the current international audio coding standards and commercial audio compression products (proprietary audio coding algorithms) [1]. Two types of the MDCT are defined, specifically, for evenly stacked [2] and oddly stacked [3] analysis/synthesis systems. In general, the evenly and oddly stacked MDCTs and MDSTs are non-invertible transforms. However, they are orthogonal in the context of complete TDAC analysis/synthesis filter banks. The analysis consists of overlapping, windowing and transforming the adjacent data blocks, while the synthesis includes inverse transforming, windowing, overlapping and adding the data blocks to perfectly reconstruct the original data sequences.

In this paper based on the matrix representation of MDCTs and MDSTs it is shown that the transposed MDCT and MDST matrices are actually the pseudoinverses of their corresponding forward transform matrices. The pseudoinverse matrix is the important concept of classical matrix theory, and though the observed relationship between the inverse MDCT/MDST and pseudoinverse seems to be trivial from linear algebra perspective, the pseudoinverse matrix and its properties provide an elegant mathematical tool to characterize MDCT/MDST as the analysis/synthesis filter banks in the matrix representation. It is important to note that recently published books [4], [5] and [6] related to the theory of lapped transforms, multirate systems and filter banks do not rigorously use the concept of pseudoinverse matrix in the context of cosine–modulated filter banks. Whereas in [6] the pseudoinverse matrix is discussed partially and it is referred to as “left-inverse”, in [4] and [5] it is not discussed at all. Essentially, all books do not contain any reference to classic theory of matrices.

2 DEFINITIONS AND PROPERTIES OF THE MDCT/MDST

2.1 The Evenly Stacked MDCT/MDST

Let $\{x_n\},\ n = 0, 1, \ldots, N-1$ represent an input data sequence. The evenly stacked MDCT (E-MDCT) and its inverse are defined as [2]

$$\begin{aligned} c_k^{E} &= \sum_{n=0}^{N-1} x_n \ \cos\left[\frac{\pi}{N}(2n+1+\frac{N}{2})\ k\right], \\ &\quad k = 0, 1, \ldots, \frac{N}{2}-1, \quad c_{\frac{N}{2}}^{E} = 0, \end{aligned} \tag{1}$$

$$\begin{aligned} \hat{x}_n^{E-MDCT} &= \frac{1}{N} \sum_{k=0}^{\frac{N}{2}-1} \epsilon_k\, c_k^{E} \ \cos\left[\frac{\pi}{N}(2n+1+\frac{N}{2})\ k\right], \\ &\quad n = 0, 1, \ldots, N-1, \end{aligned} \tag{2}$$

where $\epsilon_0 = 1$, and $\epsilon_k = 2$ for $k = 1, 2, \ldots, \frac{N}{2}-1$. The corresponding evenly stacked MDST (E-MDST) and its inverse are defined as [2]

$$\begin{aligned} s_k^{E} &= \sum_{n=0}^{N-1} x_n \ \sin\left[\frac{\pi}{N}(2n+1+\frac{N}{2})\ k\right], \\ &\quad k = 1,2,\ldots,\frac{N}{2}, \quad s_0^{E} = 0, \end{aligned} \tag{3}$$

$$\begin{aligned} \hat{x}_n^{E-MDST} &= \frac{1}{N} \sum_{k=1}^{\frac{N}{2}} \tau_k \ s_k^{E} \ \sin\left[\frac{\pi}{N}(2n+1+\frac{N}{2})\ k\right], \\ &\quad n = 0,1,\ldots,N-1, \end{aligned} \tag{4}$$

where $\tau_{\frac{N}{2}} = 1$, and $\tau_k = 2$ for $k = 1,2,\ldots,\frac{N}{2}-1$. The scaling factors ϵ_k/τ_k in the definitions of inverse E-MDCT/E-MDST are introduced for the correct matrix representation. The E-MDCT/E-MDST sequences $\{c_k^{E}\}/\{s_k^{E}\}$ possess odd antisymmetry/symmetry properties given by

$$c_{N-k}^{E} = -\ c_k^{E}, \quad s_{N-k}^{E} = s_k^{E}, \quad k = 1,\ldots,\frac{N}{2}-1, \tag{5}$$

whereas the time-domain aliased data sequences $\{\hat{x}_n^{E-MDCT}\}/\{\hat{x}_n^{E-MDST}\}$ recovered by inverse E-MDCT/E-MDST possess the following symmetries

$$\begin{aligned} &\hat{x}_n^{E-MDCT} = \hat{x}_{\frac{N}{2}-1-n}^{E-MDCT}, \qquad && \hat{x}_{n+\frac{N}{2}}^{E-MDCT} = \hat{x}_{N-1-n}^{E-MDCT}, \\ &\hat{x}_n^{E-MDST} = -\ \hat{x}_{\frac{N}{2}-1-n}^{E-MDST}, && \hat{x}_{n+\frac{N}{2}}^{E-MDST} = -\ \hat{x}_{N-1-n}^{E-MDST}, \\ &n = 0,1,\ldots,\tfrac{N}{4}-1. \end{aligned} \tag{6}$$

The properties (5) and (6) can be simply verified by proper substitution into equations (1), (2), (3) and (4).

2.2 The oddly stacked MDCT/MDST

The oddly stacked MDCT (O-MDCT) and its inverse are defined as [3]

$$\begin{aligned} c_k^{O} &= \sum_{n=0}^{N-1} x_n \ \cos\left[\frac{\pi}{2N}(2n+1+\frac{N}{2})(2k+1)\right], \\ &\quad k = 0,1,\ldots,\frac{N}{2}-1, \end{aligned} \tag{7}$$

$$\begin{aligned} \hat{x}_n^{O-MDCT} &= \frac{2}{N} \sum_{k=0}^{\frac{N}{2}-1} c_k^{O} \ \cos\left[\frac{\pi}{2N}(2n+1+\frac{N}{2})(2k+1)\right], \\ &\quad n = 0,1,\ldots,N-1. \end{aligned} \tag{8}$$

The O-MDCT is equivalent to the modulated lapped transform (MLT) [4]. The corresponding oddly stacked MDST (O-MDST) and its inverse are defined as [7, 8]

$$\begin{aligned} \overset{O}{s}_k &= \sum_{n=0}^{N-1} x_n \ \sin\left[\frac{\pi}{2N}(2n+1+\frac{N}{2})(2k+1)\right], \\ & k = 0,1,\ldots,\frac{N}{2}-1, \end{aligned} \tag{9}$$

$$\begin{aligned} \hat{x}_n^{O-MDST} &= \frac{2}{N}\sum_{k=0}^{\frac{N}{2}-1} \overset{O}{s}_k \ \sin\left[\frac{\pi}{2N}(2n+1+\frac{N}{2})(2k+1)\right], \\ & n = 0,1,\ldots,N-1. \end{aligned} \tag{10}$$

The O-MDCT and O-MDST basis functions form the modulated complex lapped transform (MCLT) [7], whose real part corresponds to the O-MDCT or MLT and imaginary part is the O-MDST. The O-MDCT/O-MDST sequences $\{\overset{O}{c}_k\}/\{\overset{O}{s}_k\}$ possess even antisymmetry/symmetry properties given by

$$\overset{O}{c}_{N-k-1} = -\,\overset{O}{c}_k, \quad \overset{O}{s}_{N-k-1} = \overset{O}{s}_k, \quad k = 0,1,\ldots,\frac{N}{2}-1, \tag{11}$$

and the time domain-aliased data sequences $\{\hat{x}_n^{O-MDCT}\}/\{\hat{x}_n^{O-MDST}\}$ recovered by inverse O-MDCT/O-MDST possess the following symmetries

$$\begin{aligned} \hat{x}_n^{O-MDCT} &= -\,\hat{x}_{\frac{N}{2}-1-n}^{O-MDCT}, & \hat{x}_{n+\frac{N}{2}}^{O-MDCT} &= \hat{x}_{N-1-n}^{O-MDCT}, \\ \hat{x}_n^{O-MDST} &= \hat{x}_{\frac{N}{2}-1-n}^{O-MDST}, & \hat{x}_{n+\frac{N}{2}}^{O-MDST} &= -\,\hat{x}_{N-1-n}^{O-MDST}, \\ n &= 0,1,\ldots,\tfrac{N}{4}-1. \end{aligned} \tag{12}$$

The properties (11) and (12) can be simply verified by proper substitution into equations (7), (8), (9) and (10).

3 THE PSEUDOINVERSE MATRIX

In this section the basic facts from classical matrix theory and computational methods of linear algebra concerning to the pseudoinverse matrix are summarized.

It is well known that if A is a real square nonsingular matrix, then there exists its unique inverse matrix denoted by A^{-1}. However, in general, if A is an $m \times n$ matrix ($m \neq n$), then the inverse matrix for A does not exist. Nevertheless, for an arbitrary $m \times n$ matrix A the pseudoinverse matrix A^+ exists, which possesses some properties of the inverse matrix [9]. The pseudoinverse matrices are directly related to the minimum norm and they have played an important role in solution of overdetermined systems of linear equations (least squares problem) [10, 11].

Let A be a real $m \times n$ matrix of rank r (matrix A has the full rank if $r = \min(m, n)$, and it is rank deficient if $r < \min(m, n)$). For an arbitrary matrix A there exists exactly one $n \times m$ matrix denoted by A^+ satisfying the four Moore-Penrose conditions [10, 11]

$$\begin{array}{ll} \text{(a)} & AA^+A = A, \\ \text{(b)} & A^+AA^+ = A^+, \\ \text{(c)} & (A^+A)^T = A^+A, \quad (A^+A)^2 = A^+A, \\ \text{(d)} & (AA^+)^T = AA^+, \quad (AA^+)^2 = AA^+. \end{array}$$

The matrix A^+ is said to be the pseudoinverse or generalized inverse of A. Conditions (c) and (d) emphasize the fact that matrices A^+A and AA^+ are Hermitian and involutory, i.e., they are symmetric and their second power is equal to the original matrix. In the case of $m \times n$ matrix A, $m > n$, if $\text{rank}(A) = n$, then the pseudoinverse matrix A^+ is given by $A^+ = (A^TA)^{-1}A^T$, while if $\text{rank}(A) = m = n$, then $A^+ = A^{-1}$. Generally, for an $m \times n$ matrix A of rank r we can perform the so-called skeleton decomposition $A = B\ C$, where B is $m \times r$ and C is $r \times n$ matrix [9]. If such a decomposition is known, then the pseudoinverse matrix A^+ can be computed from the formula

$$A^+ = C^+B^+ = C^T(CC^T)^{-1}(B^TB)^{-1}B^T.$$

The pseudoinverse matrix A^+ can be alternatively obtained via the Singular Value Decomposition (SVD) [11] or QR factorization [10]. According to the SVD for a real $m \times n$ matrix A of rank r there exists an orthogonal matrix U of order m, an orthogonal matrix V of order n, and $m \times n$ diagonal matrix Σ of rank r with positive diagonal elements that

$$U^TAV = \Sigma, \quad \Sigma = \text{diag}\,\{\sigma_1, \sigma_2, \ldots, \sigma_r, 0, \ldots, 0\},$$

where $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r > 0$ are called singular values of A. Because U and V are orthogonal matrices, $U^TU = UU^T = I$ and $V^TV = VV^T = I$, where I is the identity matrix, consequently

$$A = U\Sigma V^T.$$

It follows from the SVD decomposition that the pseudoinverse matrix A^+ is given by

$$A^+ = V\Sigma^+U^T, \quad \Sigma^+ = \text{diag}\,\{\frac{1}{\sigma_1}, \frac{1}{\sigma_2}, \ldots, \frac{1}{\sigma_r}, 0, \ldots, 0\},$$

and it satisfies the four Moore-Penrose conditions. In particular, matrices AA^+ and A^+A are given by [10]

$$AA^+ = U \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} U^T,$$

$$A^+A = V \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} V^T.$$

The computational methods for the SVD decomposition are presented in [11].

According to QR factorization of a real $m \times n$ matrix A, $m > n$, with linear independent columns there exist uniquely an $m \times n$ matrix Q and an $n \times n$ matrix R so that $Q^T\,Q$ is diagonal matrix with positive diagonal elements d_i, and R is the unit upper triangular matrix [10], that is

$$A = Q\,R, \qquad Q^T\,Q = D = \mathrm{diag}\{d_1, \ldots, d_n\}.$$

If such QR factorization of the matrix A is known, then pseudoinverse matrix A^+ is given by

$$A^+ = R^{-1}\,D^{-1}\,Q^T, \qquad Q^T\,Q = D.$$

The algorithm for computation of QR factorization is presented in [10].

4 THE MATRIX REPRESENTATIONS OF THE MDCT/MDST

An alternative method to represent the perfect reconstruction cosine-modulated filter banks is the matrix-vector notation. Consider the E-MDCT/E-MDST and O-MDCT/O-MDST defined by equations (1)/(3) and (7)/(9), respectively. The symmetry properties (5) and (11) imply that only $\frac{N}{2}$ rows of the E-MDCT/E-MDST and O-MDCT/O-MDST matrices are linear independent. Therefore, let $C^E_{\frac{N}{2}\times N}$, $S^E_{\frac{N}{2}\times N}$, $C^O_{\frac{N}{2}\times N}$ and $S^O_{\frac{N}{2}\times N}$ be the $\frac{N}{2}\times N$ E-MDCT, E-MDST, O-MDCT and O-MDST matrices, respectively. Then equations (1), (3), (7) and (9) can be written in the equivalent matrix-vector form as

$$\begin{aligned} \mathbf{c}^E &= C^E_{\frac{N}{2}\times N}\,\mathbf{x}^T, \qquad & \mathbf{s}^E &= S^E_{\frac{N}{2}\times N}\,\mathbf{x}^T, \\ \mathbf{c}^O &= C^O_{\frac{N}{2}\times N}\,\mathbf{x}^T, \qquad & \mathbf{s}^O &= S^O_{\frac{N}{2}\times N}\,\mathbf{x}^T. \end{aligned} \tag{13}$$

For the transposed E-MDCT and E-MDST matrices the following relations hold

$$C^E_{\frac{N}{2}\times N}\,[\epsilon_k\,C^E_{\frac{N}{2}\times N}]^T = S^E_{\frac{N}{2}\times N}\,[\tau_k\,S^E_{\frac{N}{2}\times N}]^T = N\,I_{\frac{N}{2}}, \tag{14}$$

$$[\epsilon_k\,C^E_{\frac{N}{2}\times N}]^T\,C^E_{\frac{N}{2}\times N} = \frac{N}{2}\begin{bmatrix} I_{\frac{N}{4}} & J_{\frac{N}{4}} & 0 & 0 \\ J_{\frac{N}{4}} & I_{\frac{N}{4}} & 0 & 0 \\ 0 & 0 & I_{\frac{N}{4}} & J_{\frac{N}{4}} \\ 0 & 0 & J_{\frac{N}{4}} & I_{\frac{N}{4}} \end{bmatrix}, \tag{15}$$

$$[\tau_k\, S^{E}_{\frac{N}{2}\times N}]^T\, S^{E}_{\frac{N}{2}\times N} = \frac{N}{2}\begin{bmatrix} I_{\frac{N}{4}} & -J_{\frac{N}{4}} & 0 & 0 \\ -J_{\frac{N}{4}} & I_{\frac{N}{4}} & 0 & 0 \\ 0 & 0 & I_{\frac{N}{4}} & -J_{\frac{N}{4}} \\ 0 & 0 & -J_{\frac{N}{4}} & I_{\frac{N}{4}} \end{bmatrix}, \tag{16}$$

whereas for the transposed O-MDCT and O-MDST matrices the following relations hold

$$C^{O}_{\frac{N}{2}\times N}\,[C^{O}_{\frac{N}{2}\times N}]^T = S^{O}_{\frac{N}{2}\times N}\,[S^{O}_{\frac{N}{2}\times N}]^T = \frac{N}{2}\, I_{\frac{N}{2}}, \tag{17}$$

$$[C^{O}_{\frac{N}{2}\times N}]^T\, C^{O}_{\frac{N}{2}\times N} = \frac{N}{4}\begin{bmatrix} I_{\frac{N}{4}} & -J_{\frac{N}{4}} & 0 & 0 \\ -J_{\frac{N}{4}} & I_{\frac{N}{4}} & 0 & 0 \\ 0 & 0 & I_{\frac{N}{4}} & J_{\frac{N}{4}} \\ 0 & 0 & J_{\frac{N}{4}} & I_{\frac{N}{4}} \end{bmatrix}, \tag{18}$$

$$[S^{O}_{\frac{N}{2}\times N}]^T\, S^{O}_{\frac{N}{2}\times N} = \frac{N}{4}\begin{bmatrix} I_{\frac{N}{4}} & J_{\frac{N}{4}} & 0 & 0 \\ J_{\frac{N}{4}} & I_{\frac{N}{4}} & 0 & 0 \\ 0 & 0 & I_{\frac{N}{4}} & -J_{\frac{N}{4}} \\ 0 & 0 & -J_{\frac{N}{4}} & I_{\frac{N}{4}} \end{bmatrix}. \tag{19}$$

where $I_{\frac{N}{4}}$ is the identity matrix, and $J_{\frac{N}{4}}$ is the reverse identity matrix, both of order $\frac{N}{4}$.

Now it is clear that the E-MDCT/E-MDST and O-MDCT/O-MDST matrices defined in equations (14)-(16) and (17)–(19) have the same full rank, i.e.,

$$\mathrm{rank}(C^{E}) = \mathrm{rank}(S^{E}) = \mathrm{rank}(C^{O}) = \mathrm{rank}(S^{O}) = \frac{N}{2}.$$

Let us denote their transposed matrices by $[C^{E}]^{+}$, $[S^{E}]^{+}$, $[C^{O}]^{+}$ and $[S^{O}]^{+}$, respectively. Using the relations (14)–(16) and (17)–(19) it can be easily verified that they satisfy Moore-Penrose conditions, and therefore they are called to be pseudoinverses of their corresponding matrices. Then both the evenly and oddly stacked inverse MDCT/MDST given by (2), (4), (8) and (10) can be written in the equivalent matrix-vector form as

$$\begin{aligned} \hat{\mathbf{x}}^{E-MDCT} &= [C^{E}]^{+}\,\mathbf{c}^{E}, \qquad & \hat{\mathbf{x}}^{E-MDST} &= [S^{E}]^{+}\,\mathbf{s}^{E}, \\ \hat{\mathbf{x}}^{O-MDCT} &= [C^{O}]^{+}\,\mathbf{c}^{O}, \qquad & \hat{\mathbf{x}}^{O-MDST} &= [S^{E}]^{+}\,\mathbf{c}^{E}. \end{aligned} \tag{20}$$

There is the following interpretation of above equations in (20) [9, 11]. If we consider the forward MDCTs/MDSTs in matrix representation given by (13) to be systems of linear equations, then time-domain aliased data sequences in (20) for given MDCT/MDST coefficients can be interpreted as least squares solutions, i.e., solutions with minimum norm. Moreover, we can derive the time-domain aliased data sequences for inverse MDCTs/MDSTs explicitly in terms of original data samples. Consider an example for $N = 8$. Multiplying both sides of equations in (13) by the corresponding transposed (pseudoinverse) MDCT/MDST matrix and using relations (15), (16), (18) and (19) we have

$$[C^{E}]^{+}\,\mathbf{c}^{E} = [C^{E}]^{+}\,C^{E}_{4\times 8}\,\mathbf{x}^{T} = [\hat{\mathbf{x}}^{E-MDCT}]^{T} = 4\left[\begin{array}{c} x_0 + x_3 \\ x_1 + x_2 \\ x_1 + x_2 \\ x_0 + x_3 \\ \hline x_4 + x_7 \\ x_5 + x_6 \\ x_5 + x_6 \\ x_4 + x_7 \end{array}\right],$$

$$[S^{E}]^{+}\,\mathbf{s}^{E} = [S^{E}]^{+}\,S^{E}_{4\times 8}\,\mathbf{x}^{T} = [\hat{\mathbf{x}}^{E-MDST}]^{T} = 4\left[\begin{array}{c} x_0 - x_3 \\ x_1 - x_2 \\ -(x_1 - x_2) \\ -(x_0 - x_3) \\ \hline x_4 - x_7 \\ x_5 - x_6 \\ -(x_5 - x_6) \\ -(x_4 - x_7) \end{array}\right],$$

$$[C^{O}]^{+}\,\mathbf{c}^{O} = [C^{O}]^{+}\,C^{O}_{4\times 8}\,\mathbf{x}^{T} = [\hat{\mathbf{x}}^{O-MDCT}]^{T} = 2\left[\begin{array}{c} x_0 - x_3 \\ x_1 - x_2 \\ -(x_1 - x_2) \\ -(x_0 - x_3) \\ \hline x_4 + x_7 \\ x_5 + x_6 \\ x_5 + x_6 \\ x_4 + x_7 \end{array}\right],$$

$$[S^{O}]^{+}\,\mathbf{s}^{O} = [S^{O}]^{+}\,S^{O}_{4\times 8}\,\mathbf{x}^{T} = [\hat{\mathbf{x}}^{O-MDST}]^{T} = 2\left[\begin{array}{c} x_0 + x_3 \\ x_1 + x_2 \\ x_1 + x_2 \\ x_0 + x_3 \\ \hline x_4 - x_7 \\ x_5 - x_6 \\ -(x_5 - x_6) \\ -(x_4 - x_7) \end{array}\right].$$

In the above equations we can clearly observe the time-domain aliased data sequences $\{\hat{x}_n\}$ recovered by the given inverse MDCT/MDST transform including both their forms in terms of original data samples and their symmetry properties originally given by equations (6) and (12).

5 CONCLUSIONS

Based on the matrix representation for the evenly and oddly stacked MDCT/MDST it has been shown that the transposed MDCT/MDST matrices are actually the pseudoinverses of their corresponding forward transform matrices. Although the observed relationship between the inverse MDCT/MDST and pseudoinverse seems to be trivial from linear algebra perspective, the pseudoinverse matrix and its properties provide an elegant mathematical tool to characterize MDCT/MDST as the analysis/synthesis filter banks in matrix representation.

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Vladimír Britaňák graduated in mathematics in 1978, received the RNDr. degree in theoretical cybernetics and mathematical informatics in 1987 from the Faculty of Mathematics and Physics of Comenius University in Bratislava, and the CSc. degree in computer science in 1995 from the Slovak Academy of Sciences in Bratislava. He is currently with the Institute of Informatics of Slovak Academy of Sciences, Bratislava, where he is a senior researcher. His research interests include image processing algorithms and digital signal processing, especially fast algorithms for discrete orthogonal transforms computation and discrete cosine/sine modulated filter banks computation.