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STRATIFIED GRAMMAR SYSTEMS WITH SIMPLE AND DYNAMICALLY ORGANIZED STRATA

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Abstract. Stratified grammar systems have been introduced as a grammatical model of M. Minsky hypothesis concerning how the mind works. This grammatical model is a restricted model since it is assumed that the strata of the mind are ordered in a given linear ordering. In this paper, we consider stratified grammar systems with strata organized dynamically, according to the current sentential form to be written, to meet Minsky hypothesis that the strata of the mind are organized dynamically according to the current task to be processed. We study the generative power of these systems, which we shall call dynamic stratified grammar systems, and we show that they generate the family of matrix grammars. Also, we consider simple systems by limiting the number of components comprising the stratum to be at most two components with only one rule each. Then, we show that every dynamic stratified grammar system can be represented by an equivalent simple one which demonstrates the ideas of generating complicated behaviors through more or less coordinated activities of entities with simpler behaviors.

Keywords: Society theory of mind; stratified grammar systems, simple systems

1 INTRODUCTION

Stratified grammar systems have been introduced in [1] as an attempt to model—at symbol level—Minsky's hypothesis concerning how the mind works [4], where mind is considered as an organized society of interrelated communicating agents grouped into agencies. When a task is to be solved, an agent takes this task and, if not succeeding to solve it, it splits the task into sub-tasks, which will be approached by agents in another stratum of the mind. The process continues until the complete solution of the task is produced. In the society model of mind [4], the strata are not necessarily predefined, but they are organized dynamically, according to the current state of the task. In [1] it is assumed that these strata are clustered (in sets of production rules), and ordered in a given linear ordering. Informally, a stratified grammar system is a system consisting of a set of strata, each stratum stratified into sets of production rules. The strata are organized in forward chaining. The generation of a string of symbols by a stratified grammar system is done in a semi-parallel manner: the passing form a stratum to another one is done sequentially according to their ordering, whereas one rule from each production set comprising the active stratum is applied in parallel to the current sentential form. In [1] it is shown that the generative power of stratified grammar systems is less powerful than that of matrix grammars.

Here we study the generative power of grammar systems with strata organized dynamically according to the current sentential form to be written, which we shall call dynamic stratified grammar systems. We show that these systems are as powerful as matrix grammars.

A basic strategy in Minsky's model is to consider as simple elements as possible in the system. Here, we limit the number of components comprising the stratum to be at most two components with only one rule each, to represent simple systems. We show that every dynamic stratified grammar system can be represented by an equivalently simple one.

2 STRATIFIED GRAMMAR SYSTEMS WITH DYNAMICALLY ORGANIZED STRATA

A *stratified grammar system* [1] *with dynamically organized strata* (of degree n, $n \geq 1$) is a construct:

$$G = (N, T, S, P_1, P_2, \dots, P_n),$$

where N is a set of nonterminals (variables), T is a set of terminals, $S \in N$ is the axiom and P_1, P_2, …, P_n are sets of sets of production rules

$$P_i = \{P_{i,1}, P_{i,2}, \dots, P_{i,k_i}\}, 1 \leq i \leq n,$$

with $k_1 = 1$ and $k_i \geq 1$, $2 \leq i \leq n$ (each $P_{i,j}$ is a set of production rules over $N \cup T$). Each P_i will be called *stratum*.

For $x, y \in (N \cup T)^*$ and for a stratum P_i, $1 \leq i \leq n$, we write $x \Rightarrow_{P_i} y$ iff all the next conditions hold [1]:

$$\begin{aligned} x &= x_1 A_{j_1} x_2 A_{j_2} x_3 \ldots x_{k_i} A_{j_{k_i}} x_{k_i+1}, \\ y &= x_1 w_{j_1} x_2 w_{j_2} x_3 \ldots x_{k_i} w_{j_{k_i}} x_{k_i+1}, \\ x_t &\in (N \cup T)^*, 1 \leq t \leq k_i + 1, \\ A_{j_r} &\rightarrow w_{j_r} \in P_{i,j_r}, 1 \leq r \leq k_i, \text{ and} \\ \{j_1, j_2, \ldots, j_{k_i}\} &= \{1, 2, \ldots, k_i\}. \end{aligned}$$

In words, from each component of the stratum P_i one rule is applied to x, in a parallel manner.

Clearly, if $|x|_N < k_i$, then the string x cannot be rewritten by the stratum P_i (blocking).

We define the language of stratified grammar system with dynamically organized strata by

$$L(G) = \{x \in T^* | \, S \Longrightarrow^*_{P_{i_1}} x_1 \Longrightarrow^*_{P_{i_2}} x_2 \Longrightarrow^*_{P_{i_3}} \ldots \Longrightarrow^*_{P_{i_t}} x_t = x, \\ t \geq 1, 1 \leq i_j \leq n, 1 \leq j \leq t\}.$$

Note that in [1] the language of stratified grammar system is defined by

$$L(G) = \{x \in T^* | \, S \Longrightarrow^*_{P_1} x_1 \Longrightarrow^*_{P_2} x_2 \Longrightarrow^*_{P_3} \ldots \Longrightarrow^*_{P_t} x_t = x, 1 \leq t \leq n\}.$$

where $\Longrightarrow^*_{P_i}$ the reflexive transitive closure of $\Longrightarrow_{P_i}$.

Now, we give some examples to illustrate the concepts.

Example 1. Let $G = (\{S, A, B\}, \{a, b\}, S,$

$$\begin{aligned} P_1 &= \{\{S \rightarrow AB\}\}, \\ P_2 &= \{\{A \rightarrow a\}, \{B \rightarrow a\}\}, \\ P_3 &= \{\{A \rightarrow b\}, \{B \rightarrow b\}\}, \\ P_4 &= \{\{A \rightarrow aA\}, \{B \rightarrow aB\}\}, \\ P_5 &= \{\{A \rightarrow bA\}, \{B \rightarrow bB\}\}). \end{aligned}$$

$$L(G) = \{xx | \, x \in \{a, b\}^* - \{\lambda\}\},$$

which is not a context-free language. (Each derivation, in the dynamic stratified grammar system G, starts with the stratum P_1 and then continues by using P_4 and/or P_5, and/or terminates by using P_2 or P_3).

Note that, in case of considering G as stratified grammar system, the derivations start with the stratum P_1 and then terminate by using P_2 obtaining the string aa.The strata P_3, P_4 and P_5 have no role. Hence, $L(G) = \{aa\}$, which is a regular language.

Example 2. Consider the following dynamic stratified grammar system
$\Gamma = (\{S, M, T, Z, N\}, \{a, b\}, S,$

$$\begin{aligned}
P_1 &= \{\{S \to MT\}\},\\
P_2 &= \{\{M \to M\}, \{T \to ZZ\}\},\\
P_3 &= \{\{N \to N\}, \{Z \to TT\}\},\\
P_4 &= \{\{M \to aN\}\},\\
P_5 &= \{\{N \to aM\}\},\\
P_6 &= \{\{N \to a\}\},\\
P_7 &= \{\{M \to a\}\},\\
P_8 &= \{\{T \to b\}\},\\
P_9 &= \{\{Z \to b\}\}).
\end{aligned}$$

Some preliminary remarks about this system are worth mentioning

- the strata P_2, P_3 double the number of occurrences of the symbols Z and T; this is possible only in the presence of the symbol M or N,
- all sentential forms contain either one occurrence of M or N, passing from M to N and vice versa can be done by using the strata P_4, P_5; this imposes the introduction of one occurrence of the terminal a,
- it is not necessary to double all occurrences of the symbols Z, T. So, the number of occurrences of Z and T is $\leq 2^k$ if k is the number of occurrences of a.

Obviously,

$$L(\Gamma) = \{a^n b^m | \, n \geq 1, 1 \leq m \leq 2^n\}$$

and this is a non-semilinear language.(Note that it is not known whether or not the family of stratified grammar systems contains non-semilinear languages; see open problem 3 in [1]).

3 THE GENERATIVE POWER

Denote by $\mathcal{L}(DSG_nCF), \mathcal{L}(SG_nCF), n \geq 1$ the families of languages generated by dynamic stratified grammar systems (of degree at most n and with context-free components) and stratified grammar systems, respectively. Also, $\mathcal{L}(DSG\ CF) = \cup_{n\geq 1}\mathcal{L}(DSG_nCF)$ and we denote by $\mathcal{L}(MAT)$ the family of matrix grammars with context-free rules and without appearance checking.

Theorem 1. $\mathcal{L}(DSG\ CF) = \mathcal{L}(MAT)$.

Proof.

(i) $\mathcal{L}(DSG_nCF) \subseteq \mathcal{L}(MAT), n \geq 1$

It is not so difficult to adapt the proof of $\mathcal{L}(SG_nCF) \subseteq \mathcal{L}(MAT), n \geq 1$ in [1] to work here. (The strata are not applied in order so eliminate the symbols $[i], 1 \leq i \leq n$).

(ii) $\mathcal{L}(MAT) \subseteq \mathcal{L}(DSG\ CF)$

Let $G' = (V_N, V_T, M, S)$ be a matrix grammar with context-free rules. Without loss of generality, we assume that G' in the 2-normal form (Lemma 1.2.3 in [2] shows how to transform a matrix grammar to its 2-normal form). Accordingly, $V_N = \{S\} \cup V_N^{(1)} \cup V_N^{(2)}, V_N^{(1)} \cap V_N^{(2)} = \phi$, and $S \notin V_N^{(1)} \cup V_N^{(2)}$, M has the form $M = M_1 \cup M_2$, where

$$m_v \in M_1 : (s_{v1}), s_{v1} : S \to AX, A \in V_N^{(1)}, X \in V_N^{(2)}, 1 \leq v \leq q,$$
$$m_t \in M_2 : (r_{t1}, r_{t2}), r_{t1} : \alpha \to \beta, \alpha \in V_N^{(1)}, \beta \in (V_N^{(1)} \cup V_T)^*,$$
$$r_{t2} : X \to Y \text{ or } X \to \lambda, X \in V_N^{(2)}, Y \in V_N^{(2)}, 1 \leq t \leq p.$$

(M_1 contains the master matrices and M_2 contains matrices with only two independent rules).

Now, we construct a dynamic stratified grammar system

$$G_1 = (V_N, V_T, S, P_1, P_2, \ldots, P_n), n = p + 1,$$

with

$$P_1 = \{s_{11}, \ldots, s_{q1}\},$$
$$P_{t+1} = \{P_{t+1,1} = \{r_{t1}\}, P_{t+1,2} = \{r_{t2}\}\}, 1 \leq t \leq p.$$

Clearly, each derivation in G' can be simulated in G_1, hence $L(G') \subseteq L(G_1)$ (the application of a matrix m_v leads to the application of the rule s_{v1} in the stratum $P_1, 1 \leq v \leq q$. Since the rules of each matrix in M_2 are independent according to the specified form of the 2-normal form, the matrix m_t in M_2, $1 \leq t \leq p$ is applicable in G' iff the stratum P_{t+1} is applicable in G_1 too).

Conversely (by similar arguments), $L(G_1) \subseteq L(G')$. Thus, $L(G) = L(G_1)$, and the theorem is proved. □

As a direct consequence of the above theorem, we have

Corollary 1. $\mathcal{L}(SG_nCF) \subseteq \mathcal{L}(DSG_mCF), m \geq n \geq 1$.

Proof. Follows from $\mathcal{L}(SG_nCF) \subseteq \mathcal{L}(MAT), n \geq 1$ [1], and the above theorem. □

4 STRATIFIED GRAMMAR SYSTEMS WITH SIMPLE AND DYNAMICALLY ORGANIZED STRATA

Most intelligent, complex systems are built from simple parts which interact in a non-simple manner such that the whole is more than the parts. In [1] two possibilities are suggested to represent simple systems. The first is to consider strata with only one component each. The second is to consider strata containing only components with only one rule each. Here, we limit the number of components comprising the stratum to be at most two components with only one rule each, to represent simple systems. Examples 1, 2 are typical examples of simple systems.

Lemma 1. For each dynamic stratified grammar system G, one can construct an equivalent simple dynamic stratified grammar system G' of the same type as G.

Proof. Let $G = (N, T, S, P)$ be a dynamic stratified grammar system with

$$P = \{P_1, P_2, \ldots, P_n\},$$
$$P_i = \{P_{i,1}, P_{i,2}, \ldots, P_{i,k_i}\}, k_i \geq 1, \text{ and } 1 \leq i \leq n.$$

We construct the simple dynamic stratified grammar system

$$G' = (V'_N, T, S', P')$$

with

$$V'_N = \{S' | \, S' \notin N\} \cup N \cup N_1 \cup \{A' | \, A \in N\} \cup \{B | \, B \notin N\}$$

where,

$$N_1 = \{(i, j) : 1 \leq i \leq n, 1 \leq j \leq k_i\},$$

S' is a new symbol (the start symbol of G'), and P' contains the following sets of strata:

(1) $S' \to S(i, 1)\}, 1 \leq i \leq n.$
(2) $\{\{A \to x'\}, \{(i, j) \to (i, j + 1)\}\}$, for $A \to x \in P_{i,j}, 1 \leq i \leq n, 1 \leq j \leq k_i - 1$, x' being obtained from x by priming all the nonterminal occurrences,
(3) $\{\{A \to x'\}, \{(i, k_i) \to (i, B)\}\}$, for $A \to x \in P_{i,k_i}, 1 \leq i \leq n, B \notin N$, x' being obtained from x by priming all the nonterminal occurrences,
(4) $\{\{A' \to A\}, \{(i, B) \to (i, B)\}\}, 1 \leq i \leq n, A \in N, B \notin N,$
(5) $\{\{A' \to A\}, \{(i, B) \to (i', 1)\}\}, 1 \leq i \leq n, 1 \leq i' \leq n, A \in N, B \notin N,$
(6) $\{\{A \to x\}, \{(i, k_i) \to \lambda\}\}$, for $A \to x \in P_{i,k_i}, 1 \leq i \leq n.$

The symbols (i, j) control the derivation steps, the primed versions of strings prevent non-parallel rewriting and the values of j (from 1 to k_i) ensure the correct using of the stratum $P_i, 1 \leq i \leq n$ (using exactly one rule from each production set comparing that stratum). The strata of type 5 allow the passing from a stratum to another one. The stratum of type 6 terminates the derivation. Therefore, $L(G') = L(G)$ and G' is of the same type as G. □

An additional simplification is limiting the number of symbols on the right hand side of each rule to be at most two (rules represent agents in Minsky's model, who asked to consider as simple agents as possible in the system).

Lemma 2. For each language in $\mathcal{L}(DSG_nCF), X \in \{CF, CF-\lambda\}, n \geq 1$, there is an equivalent simple dynamic stratified grammar system with simple rules $X \to \alpha, |\alpha| \leq 2$, generating it.

Proof. Let $G = (V_N, V_T, S, P)$ be a dynamic stratified grammar system with context-free rules. Construct an equivalent dynamic stratified grammar system

$$G' = (V_N, V_T, S, P')$$

where P' containing the strata of the form:

$$\{\{A_1 \to x_1\} \to \{A_2 \to x_2\}, \dots, \{A_{k_i} \to x_{k_i}\}\},$$
$$A_j \to x_j \in P_{i,j}, 1 \leq j \leq k_i, k_i \geq 1, 1 \leq i \leq n,$$

Clearly, $L(G') = L(G)$. □

Now consider a rule $X \to \alpha, \alpha = z_1 z_2 \dots z_s, s \geq 3, z_t \in V_{G'}, 1 \leq t \leq s$, which appears in a component of a stratum in G'. We replace this component by the components $\{X \to z_1 B\}, \{B \to z_2 B\}, \dots, \{B \to z_{s-2} B\}, \{B \to z_{s-1} z_s\}$ in the same stratum, where B is a new symbol.

Now, use Lemma 1. to construct a simple dynamic stratified grammar system G_1 equivalent to G' without priming the nonterminal B. Clearly, G_1 has the desired form, $L(G_1) = L(G')$ and hence is equivalent to G.

5 CONCLUSIONS

We introduced a grammatical model more close to M.Minsky hypothesis. Results demonstrate that intelligent, complex systems with complicated behaviors can be constructed from systems with simple elements.

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