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DETERMINATION OF FUZZY RELATIONS FOR ECONOMIC FUZZY TIME SERIES MODELS BY NEURAL NETWORKS*

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Abstract. Based on the works [11, 22, 27] a fuzzy time series model is proposed and applied to predict chaotic financial process. The general methodological framework of classical and fuzzy modelling of economic time series is considered. A complete fuzzy time series modelling approach is proposed which includes: determining and developing of fuzzy time series models, developing and calculating of fuzzy relations among the observations, calculating and interpreting the outputs. To generate fuzzy rules from data, the neural network with SCL-based product-space clustering is used.

Keywords: Fuzzy time series models, fuzzy bank rules, product-space clustering, fuzzy controllers, neural networks

1 INTRODUCTION

Much of the literature in the field of the fuzzy logic and technology is focused on dynamic processes modelling with linguistic values as its observations (see e.g. [14,

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15, 22]). Such a dynamic process is called fuzzy time series. This type of dynamic processes play very important role in making practical applications. Economic and statistical time series analysis is concerned with estimation of relationships among groups of variables, each of which is observed at a number of consecutive points in time. The relationships among these variables may be complicated. In particular, the value of each variable may depend on the values taken by many others in several previous time periods. Very often it is difficult to express exactly these dependencies, or hypothesis known for that is not there. Very frequently, in such cases more sophisticated approaches are considered. These approaches are based on the human experience knowledge and consist of series of linguistic expressions each of which takes the form of an "if ... then ..." fuzzy rule; and they are well known under the common name fuzzy controllers. But also, an expert is usually unable to linguistically describe the behaviour of economic processes in particular situations. Hence, most recent researches in the fuzzy controllers design for deriving of linguistically interpreted fuzzy rules have been focused on developing automatic methods to build these fuzzy rules using a set of numerical input-output data. To apply these methods it is supposed that a database describing previous input-output behaviour a system is available [14]. Most of these models and data-driven techniques rely on the use of Takagi-Sugeno type controllers and fuzzy/non-fuzzy neural networks [6, 8, 9, 10, 19, 24], clustering/fuzzy-clustering and genetic algorithm approaches [3, 4, 7, 11, 12, 23, 25].

The goal of this paper is to illustrate that two distinct areas, i.e. fuzzy sets theory and computational networks, may be used for economic time series modelling. This approach goes under the term "soft computing" as a synergy of methodologies useful to solve problems using some form of intelligence that differ from traditional computing. We show how to use and incorporate both fuzzy sets theory and computational networks to determine the fuzzy relational equations. As an application of the proposed method, the estimate of the inflation is carried out in this paper. The characterisation of time series is introduced in Section 2. Quantitative modelling methods of time series are presented in Sections 3 and 4. They introduce conventional and fuzzy time series modelling and show how to combine neural and fuzzy systems to produce fuzzy rules. Concluding remarks are offered in Section 5.

2 CONVENTIONAL AND FUZZY TIME SERIES

To build a time series model, in a research a sample of observations from the available data is usually collected. A time series consists of an observation set $\{y_1, y_2, \dots, y_t, \dots\}$ of some phenomenon, taken at equally spaced time intervals. We assume that y_t is real for each $t \in \mathbb{T}$, where $\mathbb{T} = \{1, 2, \dots, T\}$ is an index set. The subscript t can now be referred to as time, so y_t is the observed value of the time series at time t. The total number of observations in a time series (here T) is called the length of the time series or length of the data.

Time series models are based on the analysis of chronological sequence of observations on particular variable. The main purpose of time series analysis is to understand the underlying mechanism that generates data, and, in turn, to estimate observed data and apply the models for forecasting. In any case, time series analysis rests on the assumption that one may forecast (estimate) the value of an item by studying past movements of that item over time. Typically, in conventional time series analysis, we assume that the generating mechanism is probabilistic and that the observed series $\{y_1, y_2, \ldots, y_t, \ldots\}$ is a realisation of a stochastic process $\{Y_1, Y_2, \ldots, Y_t, \ldots\}$. This process is assumed to be stationary and is described by a class of linear models called autoregressive moving average (ARMA) models. Box and Jenkins [1] give a thorough treatment of these models. In the following, we will typically refer to realisations of stochastic processes by the notation y_t for a value at t, and $\{y_t\}$ for a full set of values corresponding to the index set $\mathbb{T} = \{1, 2, \ldots, T\}$. We will also restrict our attention to discrete stochastic process, for which the index set is a discrete set, in which case we generally use the notation y_t rather than $y(t)$, which may apply also to continuous processes.

Once an appropriate model fits, it can be used to generate forecasts for future time periods. Most forecasting methods, commonly used in time series analysis, generate forecasts of future observations that are optimal in a minimum mean, square error sense (i.e. the best linear predictor).

Next, let $\widehat{Y}_T(\tau)$ denote the forecast τ steps ahead; we define as

$$\widehat{Y}_T(\tau) = E(Y_{T+\tau} \,|\, \psi_T) \tag{1}$$

the conditional expectation of $Y_{T+\tau}$ given $\psi_T = \{Y_T, Y_{T-1}, \ldots Y_1, \ldots\}$, where E is the expectations operator and ψ_T represents a particular information set. Here we assume that we have data extending to the infinite past. Equation (1) can be used recursively to obtain the forecast values of $Y_{T+\tau}$ for $\tau = 1, 2, \ldots$ once we know the right-hand side of (1).

In practice we have a finite number of observations, the above derivation; nevertheless, on the best linear predictor in the infinite sample limit enables to develop a way of calculating the approximate best linear predictor when T is large. Recall that an invertible ARMA model can be written as $Y_t = \sum_{i=1}^{\infty} \phi_i Y_{t-i}$, where the ϕ_i weights decrease exponentially, and for large T is a good approximation to truncate the infinite sum, and truncating the infinite sum is the same as setting $Y_t = 0$ for $t \leq 0$.

In contrast to the conventional time series, the observations of fuzzy time series are fuzzy sets (the observations of conventional time series are real numbers), the universes of discourse for the fuzzy sets are subsets of $\mathbb{R}^1$, where $\mathbb{R}^1$ is the set of real numbers, either naturally or artificially defined. Song and Chisson [22] give a thorough treatment of these models. Let X_t, $(t = \ldots, 1, 2, \ldots)$, a subset of $\mathbb{R}^1$, be the universe of discourse where fuzzy sets y_t^i, $(i = 1, 2, \ldots)$ are defined and Y_t is the collection of y_t^i, $(i = 1, 2, \ldots)$. Then Y_t, $(t = \ldots, 1, 2, \ldots)$ is called a fuzzy time series on X_t, $(t = \ldots, 1, 2, \ldots)$.

3 QUANTITATIVE TIME SERIES MODELLING METHODS

Quantitative modelling methods of both conventional and fuzzy time series can be grouped into two types: time series methods and causal methods. As mentioned above, univariate time series models are based on the analysis of chronological sequence of observations on a particular variable. Causal models assume that the variable to be modelled can be explained by the behaviour of another variable, or as a set of variables.

In practice, there are many time series in which successive observations are dependent, i.e. there exists an observational relation

$$\mathrm{R} = \{(y_t,\ y_{t-1}),\ (y_{t-1},\ y_{t-2}), \ldots\} \subseteq Y_t \times Y_{t-1}, \tag{2}$$

where Y_t, Y_{t-1} denote the variables and $y_t, y_{t-1}, \ldots$ denote the observed values of Y_t and Y_{t-1} respectively. If there is a strict inclusion $\mathrm{R} \subset Y_t \times Y_{t-1}$, it is reasonable to say that variables Y_t and Y_{t-1} interact. In order to account for this interaction the usual practice is to find some analytical expression that describes this interaction.

The most often used model is, however, an explicit function

$$f:\ Y_{t-1} \to Y_t. \tag{3}$$

belonging to a prespecified class of mappings [5] (this means that we look for some relation instead of function, or for a function f such that the conditions $f(y_{t-1}) = y_t$ for $t = 1, 2, \ldots, T$ are violated. Very often the linear function (Markov process)

$$y_t = f(y_{t-1}, \phi_1, \varepsilon_t) = \phi_1 y_{t-1} + \varepsilon_t \tag{4}$$

is used, where ε_t is a random error or noise component that is drawn from a stable probability distribution with zero mean and constant variance. Equation (4) is usually called an autoregressive process of the order $p = 1$ abbreviated AR(1) because the current observation y_t is "regressed" on previous realisation y_{t-1} of the same time series. Roughly speaking, to determine the model (4) means to find the coefficient ϕ_1 such that function (3) satisfies some optimality criterion in fitting the observed data R.

The AR(1) process (4) is a special case of a stochastic process which is known as the mixed autoregressive-moving average model of order (p, q) which is abbreviated ARMA(p, q):

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \ldots - \theta_q \varepsilon_{t-q} + \varepsilon_t \tag{5}$$

where $\{\phi_1, \phi_2, \ldots, \phi_p\}$ and $\{\theta_1, \theta_2, \ldots, \theta_q\}$ are called AR coefficients and MA coefficients, respectively. As mentioned above, it is important that each invertible ARMA (p, q) process can be considered as an AR(∞) or as an approximate AR(p) model of the form

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \varepsilon_t. \tag{6}$$

If the time series of interest, say Y_t, is related to one or more other time series, then it is possible to build a model that uses the information content in these other time series. These models are called transfer function models [17] and they are logical extension of univariate ARMA models. A simple model relating the two time series $\{Y_t\}$, $\{Z_t\}$ is

$$Y_t = \psi_0 Z_t + \psi_1 Z_{t-1} + \ldots + \psi_k Z_{t-k} + e_t, \tag{7}$$

where the $\psi_1, \psi_2, \ldots, \psi_k$ are unknown coefficients and e_t is a noise component possibly assumed to be normally and independently distributed with mean zero and constant variance.

In the case of the fuzzy time series, the fuzzy relational equations can be employed as the models. Analogously to the conventional time series models, it is assumed that the observation at the time t accumulates the information of the observation at the previous times, i.e. there exists a fuzzy relation such that

$$y_t^j \ = \ y_{t-1}^i \circ R_{ij}(t,\ t-1), \tag{8}$$

where $y_t^j \in Y_t$, $y_{t-1}^i \in Y_{t-1}$, $i \in I$, $j \in J$, I and J are indices sets for Y_t and Y_{t-1} respectively, "$\circ$" is the sign for the *max-min* composition, $R_{ij}(t, t-1)$ is the fuzzy relation among the observations at t and $t-1$ times. Then Y_t is said to be caused by Y_{t-1} only, i.e.

$$y_{t-1}^i \rightarrow y_t^j \tag{9}$$

or equivalently

$$Y_t \rightarrow Y_{t-1} \tag{10}$$

and

$$Y_t = Y_{t-1} \circ R(t,\ t-1), \tag{11}$$

where $R(t, t-1)$ denotes the overall relation between Y_t and Y_{t-1}. In the fuzzy relational equation (11) the overall relation $R(t, t-1)$ is calculated as the union of fuzzy relations $R_{ij}(t,\ t-1)$, i.e. $R(t, t-1) = \bigcup_{ij} R_{ij}(t, t-1)$, where "$\bigcup$" is the union operator. In the following, we will use Mamdani's method [13] to determine these relations. For simplicity, in the following discussion, we can also express y_{t-1}^i and y_t^j as the values of membership functions for fuzzy sets y_{t-1}^i and y_t^j respectively. Since Equation (8) is equivalent to the linguistic conditional statement

$$\text{"if } y_{t-1}^i \text{ then } y_t^j\text{"}, \tag{12}$$

we have $R_{ij}(t, t-1) = y_{t-1}^i \times y_t^j$, where "$\times$" is the Cartesian product and therefore

$$R(t, t-1) = \max_{i,j}\{\min(y_{t-1}^i, y_t^j)\}. \tag{13}$$

Equation (11) is called a first-order model of the fuzzy time series Y_t with lag $p = 1$.

The first-order fuzzy time series model (11) is an important special case of the p-order model expressed by the following fuzzy relational equations

$$Y_t = (Y_{t-1} \times Y_{t-2} \times \ldots \times Y_{t-p}) \circ R_a(t, t-p) \tag{14}$$

or

$$y_t^j = (y_{t-1}^{i_1} \times y_{t-2}^{i_2} \times \ldots \times y_{t-p}^{i_p}) \circ R_a^p(t,\ t-p). \tag{15}$$

Equation (14) is equivalent to statement

$$\text{“if } y_{t-1}^{i_1} \text{ and } y_{t-2}^{i_2} \text{ and} \ldots \text{and } y_{t-p}^{i_p} \text{ then } y_t^j\text{''} \tag{16}$$

and we have

$$R_a^p(t,\ t-p) = \min_{j, i_1 \ldots i_p} \{y_t^j, y_{t-1}^{i_1}, y_{t-2}^{i_2}, \ldots, y_{t-p}^{i_p}\}, \tag{17}$$

$$R_a(t, t-p) = \max_p \left\{ \min_{j, i_1 \ldots i_p} \{y_t^j, y_{t-1}^{i_1}, y_{t-2}^{i_2}, \ldots, y_{t-p}^{i_p}\} \right\}. \tag{18}$$

We see from Equation (14) that Y_t is caused by Y_{t-1}, Y_{t-2}, ..., Y_{t-p} simultaneously.

All of the univariate fuzzy time series models discussed above can be extended to the causal (econometric) fuzzy time series models. For example, suppose that the fuzzy time series Y_t is related to a second fuzzy time series Z_{t-1} defined on universes discourse X_t and W_t, respectively, $(t = \ldots, 0, 1, 2, \ldots)$, where X_t and $W_t \in \mathbb{R}^1$. Suppose $Z_{t-1} \rightarrow Y_t$; then, analogously to the expressions (8), (9), (10), (11), we can write

$$z_{t-1}^k \rightarrow y_t^j, \tag{19}$$

$$Z_{t-1} \rightarrow Y_t, \tag{20}$$

$$y_t^j = z_{t-1}^k \circ R_{zy}(t, t-1) \tag{21}$$

or as the conditional statement “if z_{t-1}^k then y_t^j”, where $z_{t-1}^k \in Z_{t-1}$, $(k = 1, 2, \ldots)$, $y_t^j \in Y_t$ are fuzzy sets, $k \in K$, where K is the index set for Z_{t-1}.

4 DETERMINATION OF FUZZY RELATIONS BY NEURAL NETWORKS

All the above fuzzy time series models can be determined if in particular models the fuzzy relations are known. Since finding the exact solution of fuzzy relations is generally very difficult and unrealistic in practice, more sophisticated approaches are considered very frequently. All these approaches are well known under the common name “fuzzy controllers”. Now, we will illustrate how to obtain the fuzzy rules of the type of (12) or (16). Basically there are three applicable methods for deriving these rules [16]:

1. Methods based on human aperator's experience,

2. Methods based on a modelling of human operator's controlling actions,
3. Methods based on a model of a process.

Since first two methods are frequently used in technical or technological systems, we will apply the third method. As mentioned earlier, to apply the third method it is supposed that a database describing previous input-output behaviour of a system and the adequate model of the observed process are available.

In a fuzzy system, neural networks represent a powerful tool for generating fuzzy rules purely from data. Neural networks can adaptively generate the fuzzy rules in a fuzzy system by SCL-based product-space clustering technique [11]. Next, in a numerical example, we will illustrate and show, how to obtain fuzzy rules using the fuzzy sets theory and neural networks.

Let us consider a simple example. The data set used in this example (the 514 monthly inflations in the U.S. for the forty-three years 1956–1998) was published at `http://neatideas.com/data/inflatdata.htm`. As many financial time series, the original data exhibit considerable inequalities of the variance over time, and the log transformation stabilises this behaviour. Figure 1 illustrates the time plot of this time series. This time series shows no apparent trend or periodic structure. We would like to develop a time series model for this process so that a predictor for the process output can be developed. To build a forecast model the sample period for analysis $y_1, \ldots, y_{344}$ was defined, i.e. the period over which the forecasting model can be developed and estimated, and the ex post forecast period (validation data set), $y_{345}, \ldots, y_{514}$ as the time period from the first observation after the end of the sample period to the most recent observation. By using only the actual and forecast values within the ex post forecasting period only, the accuracy of the model can be calculated.

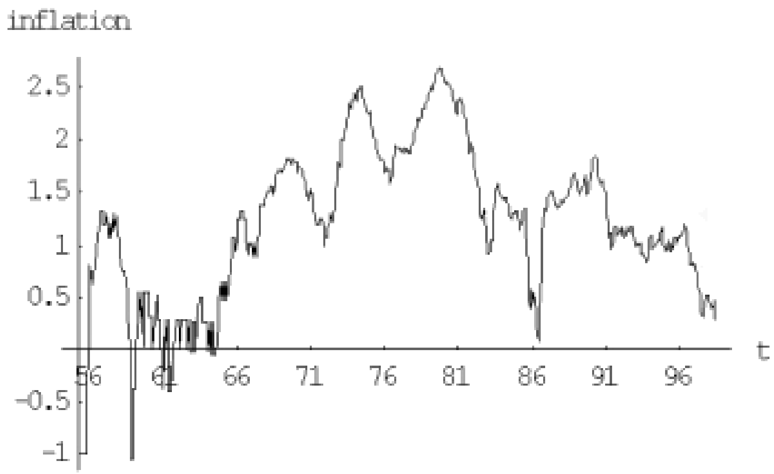

Fig. 1. Natural logarithm of monthly inflation from February 1956 to November 1998

Input selection and data preprocessing are of crucial importance to the development of time series models. Potential inputs (independent variables) were chosen based on traditional statistical tools. These include the autocorrelation function (ACF), the partial autocorrelation function (PACF) and the Akaike Information Criterion (AIC) [26]. Following this, at the starting point we have formulated a model relating the value y_t of the series at time t that depends only on its previous value y_{t-1} and on the random disturbance ε_t, i.e.

$$y_t = \xi + \phi_1 y_{t-1} + \varepsilon_t, \tag{22}$$

where the variable y_t (in our case the first inflation rate difference) is explaned by its previous values only, and ε_t is a white noise disturbance term. Using Levinson-Durbin algorithm [2, 18] the model (22) is statistically fitted as

$$\widehat{y}_t = -\,0,1248 y_{t-1}. \tag{23}$$

In input selection, all the above techniques based on the traditional statistical analysis are in fact imprecise (the theoretical ACF was estimated by the sample ACF, etc.). In fact we obtain a certain number of input values, but we are sure that these values are one of many other possible values. Thus, we will further suppose that the potential inputs, which were chosen based on statistical analysis, are crisp data. Sometimes it may be advantageous to convert them into fuzzy sets (linguistic values) characterized by membership functions (the uncertainty is modelled as a possibility distribution). At this stage, we will only give some outlines to model a fuzzy time series in a fuzzy environment. The fuzzy time series modelling procedure consists of an implementation of several steps, usually as follows:

1. Define the input-output variables and the universes of discourse.
2. Define (collect) linguistic values and fuzzy sets on the universes of discourse.
3. Define (find) fuzzy relations (fuzzy rules).
4. Apply the input to the model and compute the output.
5. Defuzzify the output of the model.

The proposed fuzzy time series modelling procedure is divided into five steps. From Step 1 to Step 2, the input data are fuzzified, in Step 3, analogously to the conventional model (22), the fuzzy time series model, i.e. the fuzzy relational model is created. Steps 4, 5 are considered as an application of the model (i.e., analysis of economic structures and the forecasting). In the literature this modelling approach is known as the fuzzy rule based system (see Figure 2). Below we will discuss these steps and apply them to the inflation time series at a more detailed level. In Figure 2 the fuzzy rule based system has three blocks: (a) block for fuzzification of input variables, (b) knowledge base block, and (c) defuzzification block.

Firstly, in the fuzzification block, we specified input and output variables. The input variable x_{t-1} is the lagged first difference of inflation values $\{y_t\}$ and is calculated as $x_{t-1} = y_{t-1} - y_{t-2}$, $t = 3, 4, \ldots$. The output variable x_t is the first difference

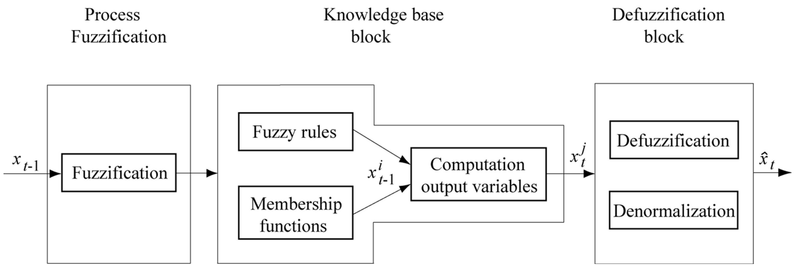

Fig. 2. Structure of the fuzzy system for inflation forecasts

of inflation values $\{y_t\}$ and it is calculated as $x_t = y_t - y_{t-1}$, $t = 2, 3, \ldots$. The variable ranges are as follows:

$$-0,75 \leq x_t, x_{t-1} \leq 0,75.$$

These ranges define the universe of discourse within which the data of x_{t-1} and x_t are, and on which the fuzzy sets have to be, specified. The universes of discourse were divided into the seven intervals.

Next, we specified the fuzzy-set values of the input and output fuzzy variables. The fuzzy sets numerically represented linguistic terms. Each fuzzy variable assumed seven fuzzy-set values as follows: NL: Negative Large, NM: Negative Medium, NS: Negative Small, Z: Zero, PS: Positive Small, PM: Positive Medium, PL: Positive Large.

Fuzzy sets contain elements with degrees of membership. Fuzzy membership functions can have different shapes. The triangular membership functions were chosen. Figure 3 shows membership function graphs of the above fuzzy sets.

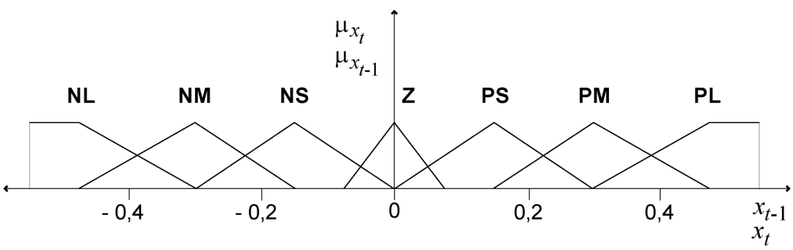

Fig. 3. Fuzzy membership functions of fuzzy variables x_{t-1} and x_t

The input and output spaces were divided into the seven disjoint fuzzy sets. From membership function graphs μ_{t-1}, μ_t Figure 3 shows that the seven intervals $[-0,75; -0,375]$, $[-0,375; -0,225]$, $[-0,225; -0,075]$, $[-0,075; 0,075]$, $[0,075; 0,225]$, $[0,225; 0,375]$, $[0,375; 0,75]$ correspond to NL, NM, NS, Z, PS, PM, PL, respectively.

Next, we specified the fuzzy rule base or the fuzzy relations bank. The appendix describes the neural network which uses the supervised competitive learning to derive fuzzy rules from data. As shown in Figure 4(b) the bank contains the 5 fuzzy rules. For example, the fuzzy rule of the 34$^{\text{th}}$ block corresponds to the following fuzzy relation

$$\text{IF } x^i_{t-1} = \text{PM THEN } x^j_t = \text{PS}. \tag{24}$$

Finally, we determined the output action given the input conditions. The Mamdani's implication [13] was used. Each fuzzy rule produces the output fuzzy set clipped at the degree of membership determined by the input condition and the fuzzy rule. When the input value, say $x^i_{t-1} = x^i_{344}$, is applied to the model (8), the output value $x^j_t = x^j_{345}$ can be calculated. It is possible to compute the output fuzzy value x^j_t by the following simple procedure consisting of three steps:

- Compute the membership function values $\mu_{\text{NL}}(x_{t-1}), \mu_{\text{NM}}(x_{t-1}), \ldots, \mu_{\text{PL}}(x_{t-1})$ for the input x_{t-1} using the membership functions shown in Figure 3.
- Substitute the computed membership function values in fuzzy relations (12), (24).
- Apply the *max-min* composition to obtain the resulting value x^j_t of fuzzy relations.

Following the above principles, we have obtained the predicted fuzzy value for the inflation $x_t = x^j_{345} = 0,74933$.

The inflation values in the output x^j_t, $t = 345, 346, \ldots$ are not very appropriate for a decision support because they are fuzzy sets. To obtain a simple numerical value in the output universe of discourse, a conversion of the fuzzy output is needed. This step is called defuzzification. The simplest defuzzification scheme seeks for the value $\widehat{x}_t$ that is of middle membership in the output fuzzy set. Hence, this defuzzification method is called the Middle of Maxima, abbreviated MOM. Following this method, we have obtained the predicted value for the $\widehat{x}_{345} = -0,15$. The remaining forecasts for ex post forecast period $t = 346, 347, \ldots$ may be generated in a similar way.

As a final point, let us examine what has been gained by use of a fuzzy time series model over an ordinary AR(1) model for the output x_{345}. For this purpose, we have computed prediction limits on the one-step-ahead forecast from the AR(1) model, and fuzzy time series model. The 95 percent interval around the actual inflation value based on the statistical theory is $\widehat{x}_{345} \mp u_{1-\frac{\alpha}{2}} \widehat{\sigma}_\varepsilon (1 + \phi_1^2)^{\frac{1}{2}} = 0,00312 \mp 1,96\ 0,15476(1 + (-0,1248)^2)^{\frac{1}{2}} = (-0,0442; 0,05043)$, where $\widehat{x}_{345}$ represents the forecast for period $t = 345$ made at origin $t = 344$, $u_{1-\frac{\alpha}{2}}$ is a $100(1-\frac{\alpha}{2})$ percentage of the standard normal distribution, and $\widehat{\sigma}_\varepsilon$ an estimate of the standard deviation of the noise. An intuitive method for constructing confidence intervals for fuzzy time series model is simply the defuzzification method First of Maxima and First of Minima to obtain prediction limits on the one-step-ahead forecast. In our example, the "confidence" interval for fuzzy time series value $\widehat{x}_{345} = 0,00312$ is

($-0,30256$ to $0,3088$). The actual value for the AR(1) model does not fall within the forecast interval, and moreover, its sign is opposite to the forecast value sign.

5 CONCLUSION

In this paper, we have presented an application of the fuzzy time series model to forecast an autoregressive process. A formal framework for the definition of fuzzy rules has been given. This framework is based on the simple competitive learning of networks. The neural network with the SCL clustering technique was used to determine the fuzzy relation (fuzzy rules) of first-order fuzzy time series models directly from data. The proposed method is also suitable for parameter estimations of econometric models, in applications of deterministic non-linear dynamics and chaos theory in contemporary economics and finance.

In comparison of proposed techniques with statistical approaches, the AR(1) model generates worse one-step-ahead forecasts. Furthermore, pure statistical models will involve greater computational effort, and will be more difficult to modify.

Because the results were based of chosen inflation rates and data, they were difficult to generalize in others situations. Yet, the results certainly provide a rational way for improvement of forecasting abilities in chaotic economic systems.

The method may be of real usefulness in practical applications, where the expert usually cannot explain linguistically what control actions the process takes or there is no knowledge of the process. In principle a neural network can derive this knowledge from data. In practice this is usually necessary. Although the method has been carried out in the time series modelling field, it is suitable for other applications as data mining systems, information access systems, etc.

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APPENDIX

GENERATING FUZZY RULES BY SCL-BASED PRODUCT-SPACE CLUSTERING

The neural network shown in Figure 4 was used to generate structured knowledge of the form "if A, then B" from a set of numerical input-output data. In Section 4 we defined cell edges with the non-overlapping seven intervals of the fuzzy-set values in Figure 3. The interval $-0,75 \leq x_t,\ x_{t-1} \leq 0,75$ was partitioned into seven non-uniform subintervals that represented the seven fuzzy-set values NL, NM, NS, Z, PS, PM, and PL assumed by fuzzy variables x_{t-1} and x_t. The Cartesian product of these subsets defines $7 \times 7 = 49$ fuzzy cells in the input-output product space $\mathbb{R}^2$. As mentioned in [10] these fuzzy cells equal fuzzy rules. Thus, there are total 49 possible rules and thus 49 possible fuzzy relations.

We can represent all possible fuzzy rules as 7-by-7 linguistic matrix (see Figure 5). The idea is to categorise a given set or distribution of input vectors $\mathbf{x}_t = (x_{t-1}, x_t)$, $t = 1,\ 2, \ldots,\ 344$ into $7 \times 7 = 49$ classes, and then represent any vector just by the class into which it falls.

We used SCL (Supervised Competitive Learning) [10, 14] to train the neural network in Figure 4. The software was developed at tha Institute of Computer Science of the Faculty of Philosophy and Science, Opava. We used 49 synoptic quantization vectors. For each random input sample $\mathbf{x}_t = (x_{t-1}, x_t)$, the winning vector $\mathbf{w}_i = (w_{1i'}, w_{2i'})$ was updated by the SCL algorithm according to

$$\left.\begin{aligned}\widetilde{w}_{1i'} &\leftarrow \widetilde{w}_{1i} + \eta(\widetilde{x}_{1t} - \widetilde{w}_{1i})\\ \widetilde{w}_{2i'} &\leftarrow \widetilde{w}_{2i} + \eta(\widetilde{x}_{2t} - \widetilde{w}_{2i})\end{aligned}\right\} \quad if\ i = i',$$

$$\left.\begin{aligned}\widetilde{w}_{1i'} &\leftarrow \widetilde{w}_{1i} - \eta(\widetilde{x}_{1t} - \widetilde{w}_{1i})\\ \widetilde{w}_{2i'} &\leftarrow \widetilde{w}_{2i} - \eta(\widetilde{x}_{2t} - \widetilde{w}_{2i})\end{aligned}\right\} \quad if\ i \neq i',$$

where i' is the winning unit defined $\|\widetilde{\mathbf{w}}_{i'} - \widetilde{\mathbf{x}}_t\| \leq \|\widetilde{\mathbf{w}}_i - \widetilde{\mathbf{x}}_t\|$ for all i, and where $\widetilde{\mathbf{w}}_i$ and $\widetilde{\mathbf{x}}_t$ are normalized versions of $\mathbf{w}_i$ and $\mathbf{x}_t$, respectively, and η is the learning coefficient.

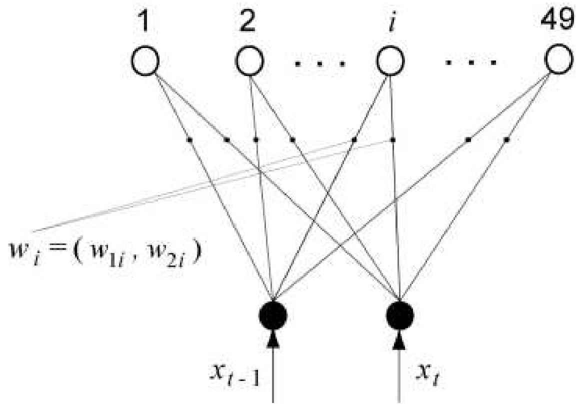

Fig. 4. The topology of the network for fuzzy rules generating by SCL-based product-space clustering

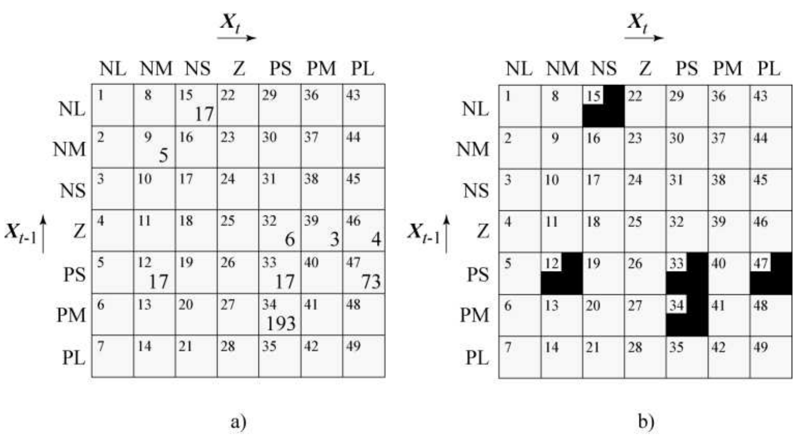

Fig. 5. Distribution of input-output data (x_{t-1}, x_t) in the input-output product space $X_{t-1} \times X_t$ (a). Bank of fuzzy rules of the time series modelling system (b).

Supervised Competitive learning (SCL)-based product-space clustering classified each of the 344 input-output data vectors into 9 of the 49 cells as shown in Figure 5 (a). Figure 5 (b) shows the fuzzy rule bank. We added a rule to the rule bank if the count of input-output vectors in particular cells was larger than the value 0,05N, where $N = 344$ is number of data pairs (x_{t-1}, x_t), $t = 1, 2, \ldots, N$ in the input and output series. For example, the most frequent rule represents the cell 34. From most to least important (frequent) the fuzzy rules are (PM; PS), (PS; PL), (NL; NS), (PS; PL), and (PS; PS).