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REGISTRATION OF HOLOGRAPHIC IMAGES BASED ON INTEGRAL TRANSFORMATION

Pavol Božek

Institute of Applied Informatics, Automation and Mathematics
Faculty of Materials Science and Technology
Slovak University of Technology
Hajdóczyho 1, 917 24 Trnava, Slovakia
e-mail: pavol.bozek@stuba.sk

Elena Pivarčiová

Department of Informatics and Automation Technology
Faculty of Environmental and Manufacturing Technology
Technical University in Zvolen
Masarykova 24, 960 53 Zvolen, Slovakia
e-mail: pivarciova@tuzvo.sk

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Abstract. The paper describes the possibilities of using Fourier-Mellin transform for registering images of holographic interferograms. Registered holographic images will then allow automating their evaluation. Registration based on changes in image intensities using the discrete integral transforms was selected from the methods of registration. Whereas it was necessary to register the images, which are not only translated, but also rotated and with the change of scale, the Fourier-Mellin transform was used. Use of the image discrete transforms is original in this field, the proposed processing algorithm also contains simplified mean of calculating the angle of rotation of the test image instead of common Fourier-Mellin transformation method sequence.

Keywords: Registration of images, Fourier-Mellin transformation, holographic image, integral transformation

1 INTRODUCTION

Registration of images enables maximum possible geometric alignment of corresponding points and objects in the pictures, it means identifying with position and spatial orientation of these objects. Found geometric transformation transforms registered images into a common coordinate space [14].

Registration of images is used for geometric alignment of two or more digital images, which represent the view of the same object, which can be obtained by various methods, sensors or from different directions of view or by the same sensor but in different times. The compared images may have different intensity and geometry.

Registration of images is used for different purposes [14, 10]:

- Connecting the information obtained by various procedures – e.g. registration of data obtained by a combination of several sensors. In medicine, the registration of morphological data obtained by computer tomography, nuclear magnetic resonance, one-photon emission tomography, positron emission tomography were used. Registration of medical images from several modalities is subject to more thorough visual examination and allows getting more comprehensive information on the structure and function of different organs.
- Tracking changes – comparison of images of the same object obtained at different times or at different conditions. From such justified slides we can then evaluate the change of size or shape.
- Object comparison and recognition – for example the verification of the signature, segmentation (breakdown) of objects.
- Getting the panoramic image of a series of partially overlapping successive frames.
- The improvement of the quality of the background noise of degraded image data of the same scene, which is necessary to register before using averaging techniques.
- Processing of remotely scanned data, such as the registration of satellite imagery.

For registration of images various methods can be used. Most common methods of registration include:

- Cross-correlation based on the correlation coefficient or the sum of the absolute values of the differences of the criterion of differences.
- Comparison based on phase-only matching of Fourier transform.
- Using point fitting.
- Stochastic sign change criterion.
- Method of least-squares fittings on matched point pairs.
- Moment-based registration method.
- Mutual information method.

- Using Fourier-Mellin transformation, invariant to translation, rotation and change of scale of the image.

The Fourier-Mellin transformation can also be used for the registration of images, watermarks, invariant pattern recognition, preprocessing of images.

Fourier-Mellin transformation and registration of images take up the work of multiple authors. [10] is devoted to the registration of medical images using Fourier-Mellin transformation. [22] have done the registrations of images which have been shifted, rotated and have modified scale. [8] focuses on Fourier's transformation defined on 2D and 3D. [17] was looking for the invariant content of images in multimedia archives using Fourier-Mellin transformation. [21] used the Fourier-Mellin transformation for the comparison of plant leaves. [24] suggested some Fourier's transformation for the subgroups, including tapering transforms. [9] took up detection of duplicate images in large databases. [3] used the approximation of Fourier-Mellin transformation for the reconstruction of the grayscale images. [16] used the Fourier-Mellin transform for detecting watermark in images regardless of the scaling and rotation. [18] detected human face. [20] used this method for comparing of distorted objects. [5] have used the registration of images for processing the photos from the full Eclipse of the Sun. [26] registered the images using log-polar transformation. [19] treats the panoramic images. [12] used the Fourier-Mellin transformation in radiation therapy.

Incomplete algorithms of Fourier-Mellin transformation are shown in [27, 29]. Implementation of the Fourier-Mellin transformation algorithm is reported in [25, 15].

Use of the registration of images for processing holographic interferograms is not described in any of the available works.

Registration of images was used for holographic images obtained using holographic interferometry [2]. Figure 1 shows demonstrations of holographic interferograms of concentration fields. Holographic images differ by the number and position of interference stripes, which need to be analyzed and evaluated.

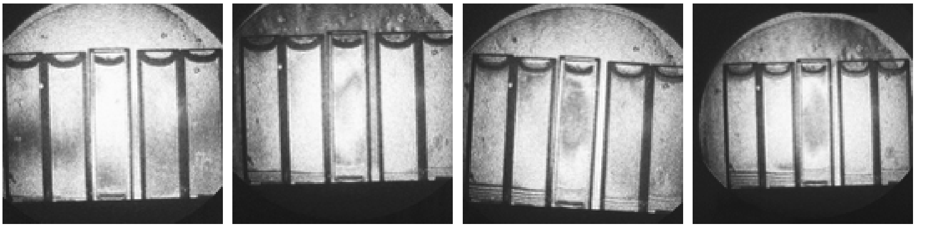


Fig. 1. Images of holographic interferograms of concentration fields

Currently, the use of holographic interferometry as quantitative measuring and contactless diagnostic method in basic and application research largely depends on the process of evaluation of interferograms. Since this is a tedious, time-consuming process, with a large number of gained holographic images, it is necessary to replace the entire process by automatic calculation. One of the most difficult steps in computer processing of holographic images is determining the positions of interference stripes and determine the layout of the interference orders. The determination of the refractive index and calculation of the required variables (e.g. temperature or concentrations) will then be made.

Whereas when processing holographic images of experiments there are usually a large number of images (of the order of several hundreds), which is necessary to process and assess so that it is possible to automate this process [7, 13], the whole set of holographic images must have the same position, dimensions, role, and alignment.

Solving the problem is to transform the position of holographic images (registration of images), whose task is to remove the mutual displacement, rotation and zooming of images and prepare them for subsequent evaluation. The aim is to make the whole set of holographic images to be registered.

2 REGISTRATION OF IMAGES

We have the reference image $a(x)$ and the input image $b(x)$, which has to be the identical with the reference image. The registration function of geometric transformation is to be estimated from the similarity of the characteristics of these images.

Consider that the image of $b(x)$ is the displaced copy of image $a(x)$:

$$b(x) = a(x - x_0). \tag{1}$$

Their Fourier's transformation $A(u)$ and $B(u)$ have the relationship:

$$B(u) = \mathrm{e}^{-j2\pi u^t x_0} A(u). \tag{2}$$

We can construct a correlation function [28] as [10]:

$$Q_p(u) = \frac{A^*(u)}{|A(u)|} \cdot \frac{B(u)}{|B(u)|} = \mathrm{e}^{j(\theta_b(u) - \theta_a(u))}, \tag{3}$$

where $\theta_a(u)$ and $\theta_b(u)$ are phases of $A(u)$ and $B(u)$.

In the absence of noise, this function can be expressed in the form:

$$Q_p(u) = \mathrm{e}^{-j2\pi\left(u^t x_0\right)}. \tag{4}$$

Its inverse Fourier transform is Dirac δ-function centered in [10]:

$$u = m_0 = [x_0, y_0]^t. \tag{5}$$

Registration is accomplished by detecting the occurrence of Dirac δ-function in the inverse transformation of function $Q_p(u)$. The coordinates of the maximum culmination of δ determine the image translation.

In practice the noise in the image $Q_p(u)$ can be complicated by the search for global maximum [5]. Therefore, it is advantageous to use the low pass filter, the weight function which "mutes" high frequencies (noise). The result is a matrix that has a clear peak, whose position (deviation from the center) corresponds to mutual displacement of the images.

The disadvantage of this method is that without further adjustment it is not possible to register other transformation than the shifts. When registering holographic images it is necessary to synchronize the images not only to each other but also images rotated, or with modified scale.

Modification of the above method – use of the Fourier-Mellin transformation – allows registration of shifted or rotated images and with different scaling.

Fourier-Mellin transformation combines aspects of the Fourier and Mellin transformation with the transformation into log-polar coordinates of the image.

Registration of images using Fourier-Mellin transformation uses phase and amplitude. This method uses the fact that the differences of shifts are ignored, because the amplitude spectrum of the image and its displaced copy is identical, only their phase spectrum varies.

The rotation can be converted to shift transformation of images into a polar coordinate system. However, we need to know the center of rotation, which, of course, is unknown in practice. This problem can be eliminated by working with the amplitude spectrums of images [23].

If image b is rotated by an angle with respect to image a, the amplitude spectrum $|(F)(b)|$ against the spectrum $|F(a)|$ is rotated by about the same angle. However, in this case the center of rotation is known – it is the point representing the zero frequency.

If the amplitude spectrums $|F(a)|$, $|(F)(b)|$ are transformed to the log-polar coordinate system (the spectrum is converted to polar coordinates and the distance from the origin of the coordinate system to logarithmic scale), using the above-described phase correlation method we identify not only the rotation, but also the scale change.

Fourier-Mellin transformation converts the rotation and zooming to easy shifts in the parametric space and allows the use of the phase correlation techniques. Phase correlation can then be used to determine the angle of rotation and scale between the pair of images [26].

Image function $f(x, y)$ may be sampled as a function $f(\theta, e^r) = f(\theta, \rho)$, where r is the distance from the center of the image (see Figure 2).

Suppose that the centre of the image is the starting point for the transformation. Each pixel in the image can be represented as the distance r from the center of the image and the angle θ. If we rotate the image, only θ is changed, r remains the same.

If we use the exponential scale $\log r$ instead of a representation of the second pixel coordinate as the amount of r, we can convert the change of scale to translation [11].

If the image has been resized to scale according to k, the Cartesian point $P(x, y)$ in the image will be in log-polar coordinates represented as $P(\theta, \log(kr))$. Then the point P with the changed scale will be expressed as translation: $P(\theta, \log k + \log r)$ [21].

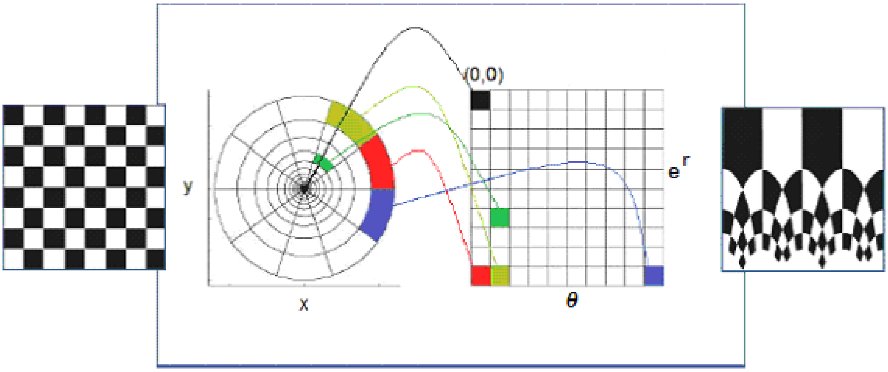


Fig. 2. Log-polar transformation [6]

2.1 Used Conversion from Cartesian to the Log-Polar Coordinates

a) Log-polar transformation of the amplitudes $|A(u,v)|, |B(u,v)|$ from Cartesian to the log-polar coordinate system

Fourier transform is displayed in log-polar plane by transformation of the coordinates (Figure 3). We use $r = \sqrt{x^2 + y^2}$ for the conversion from Cartesian to the log-polar coordinates.

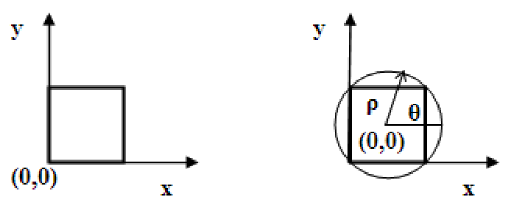


Fig. 3. Transformation from rectangular to polar coordinates by [4]

The origin (m_0, n_0) should be in the middle of the image matrix, to ensure the maximum number of pixels. If the image is formed by a square grid of $N \times N$ points, the coordinates of the center will be:

$$m_0 = n_0 = \begin{cases} N/2 & \text{if } N \text{ is odd,} \\ (N-1)/2 & \text{if } N \text{ is even.} \end{cases} \tag{6}$$

Maximum sampling radius for conversion will be:

$$\rho_{\max} = \begin{cases} \min(m_0, n_0) & \text{inscribed circle} \\ \sqrt{m_0^2, n_0^2} & \text{described circle.} \end{cases} \tag{7}$$

If the inscribed circle is selected as the limit for conversion, some pixels which lie outside of the circle will be ignored. If described circle is selected, all the pixels will be included, defective pixels however (pixels inside the circle, but outside the picture matrix) will be included as well. Whereas the pixels in Cartesian coordinates cannot be mapped one to one to the log-polar coordinates, the average of surrounding pixels (nearest neighbor, bilinear or bicubic downsampling) must be calculated.

The relationship between polar coordinates (ρ, θ), which is sampling the input image to the log-polar image (e^r, θ) is given by:

$$(\rho, \theta) = (e^r, \theta). \tag{8}$$

For pixel mapping from the input image (x_i, y_i) to pixels of the output image (r_m, θ_n) the following applies [4]:

$$\begin{aligned} x_i &= \mathrm{round}(\rho_m \cdot \cos(\theta_n) + m_0)\,, \\ y_j &= \mathrm{round}(\rho_m \cdot \sin(\theta_n) + n_0)\,, \end{aligned} \tag{9}$$

where $(\rho_m, \theta_n) = (e^{rm}, \theta_n)$ by (8). The input image is of dimension $i \times j$ and the output image is of dimension $m \times n$.

b) Fourier transformation of log-polar amplitudes

$$\begin{aligned} A_{lp}(\nu, \varpi) &= \mathcal{F}\left\{|A_{lp}\left(e^r, \theta\right)|\right\}, \\ B_{lp}(\nu, \varpi) &= \mathcal{F}\left\{|B_{lp}\left(e^r, \theta\right)|\right\}. \end{aligned} \tag{10}$$

Log-polar transformation of amplitude spectrum causes the rotation and scaling to arise as the shift. It is therefore possible to use the phase correlation to detect the angle of rotation and scale between the pair of images.

Using phase correlation of the results of the Fourier-Mellin transformation A_{lp}, B_{lp}, we find the rotation size and scale of the test image b against a reference image a. By backward rotation and scaling the test image b we create image b'. Then we calculate the Fourier transformation of the image b' and the reference image a. Using phase correlation we calculate displacement of images. Backward shift of the image b' creates image b''.

2.2 The Designed Algorithm of Registration of Images Using Fourier-Mellin Transformation

1. Load of the Img1 (reference) and Img2 (input)

   [1a.] Preprocessing of input images from (1)
   Locate areas of interest in the image, and move to the center of the image

   [1b.] Hamming window for input images from (1) or (1a).

2. Calculation of Fast Fourier transform (FFT) for Img1 and Img2 from (1) or (1a), (1b)
   - 2a. Extraction of amplitudes from (2).
3. The transformation of the amplitudes from (2a) to the log-polar coordinates (using bicubic interpolation).
4. Calculation of the FFT for amplitude from (3)
   - 4a. Amplitude extraction from (4)
   - 4b. Phase extraction from (4).
5. Phase correlation for SCALING and ROTATION from the phases (4b) [Gaussian low pass filter].
6. The detection of maxima dX, dY of the phase correlation from (5).

— REGISTRATION —

7. Calculation of the scale ρ from the value of dY from (6).
8. Calculate the angle of rotation θ from the value of dX from (6).
9. Backward rotation and scale change of the test image (using bicubic interpolation)
   - 9a. Change of rotation of Img2 from (1) of angle: − rotation from (8)
   - 9b. Change of the scale of the backward rotated Img2 from (9a) of 1/scale from (7)
   - 9c. Complementing or trimming the size of modified Img2 from (9b) to the size of Img1 from (1).

[10.] Hamming window for registered Img1 from (1) and Img2 from (9c).

11. Calculation of the FFT for registered images from (10)
    - 11a. Extraction of phases from (11).
12. Phase correlation for the SHIFT of phases (11a) [Gaussian low pass filter].
13. Calculation of displacement Δx, Δy as the deviation of the maxima of the phase correlation of (12) from the center of the correlation matrix.
14. Backward shift of Img2 from (9c) according to (13).
15. Output of parameters of image transformations $(\Delta x, \Delta y, \rho, \theta)$.

— End of REGISTRATION —

16. Cutting of the effective area of registered images.

Note: In square brackets [ ] optional parametric parts of the algorithm are given.

Figures 4 a) and 4 b) show the correlation phase demos for the calculation of rotation and scaling and of displacement of the two images.

Calculation of ΔX, ΔY as the deviation of the maxima of the phase correlation from the center of the correlation matrix:

```
dX = centerS - CenterMaxX + 1
dY = centerR - CenterMaxY + 1
```

The algorithm of calculation of the scale from the values of ΔY:

```
if dY == 0
    scale = 1
else
    if dY < 0
        scale = (rho(-dY + 1) + rho(-dY + 2))/2
    else
        scale = 2/(rho(dY + 1) + rho(dY + 2))
    end
end
```

where `rho` is the vector for the calculation of the scale (see below).

Calculation of the angle of rotation from the value of ΔX:

```
degToPix = 360/(numC + 1);
rotation = -degToPix * (dX)
```

where `numC` is the number of columns of the test image.

For the calculation of the scale of the ρ the vector with the logarithmic layout between $\langle \log 10, \log 10^d \rangle$ was used:

```
d = min([Ac-Center(1) Center(1)-1 Ar-Center(2) Center(2)-1]);
rho = logspace(log10(1), log10(d), Nrho);
```

where `Nrho` is the number of points-lines of the transformed image, `Ac` is the number of columns of the input image, `Ar` is the number of rows of the input image, `Center` is the center of the input image.

For the calculation of the angle of rotation θ the vector with linear distributed points of the interval $\langle 0, 2\pi \rangle$ was used:

```
theta = linspace(0, 2*pi, Ntheta+1);
```

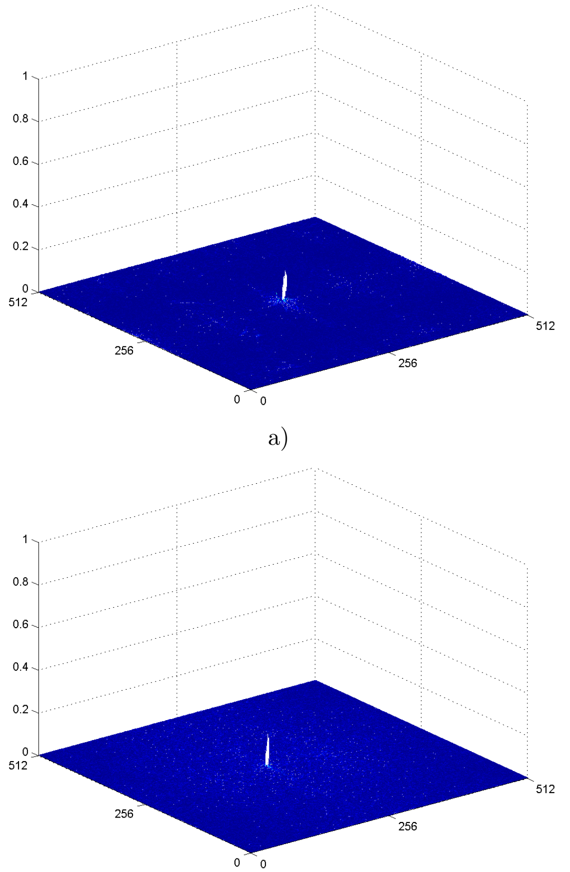
where `Ntheta+1` is the number of points – columns of the transformed image.

For the conversion of the polar coordinates to Cartesian coordinates we use the known equations:

```
xx = rho*cos(theta) + Center(1);
yy = rho*sin(theta) + Center(2);
```

3 USING FOURIER-MELLIN TRANSFORMATION FOR THE REGISTRATION OF HOLOGRAPHIC IMAGES

Registration using Fourier-Mellin transformation was applied to the set of holographic images, obtained during the experiment. The kit contained delayed, re-



a)

b)

Fig. 4. Phase correlation for the calculation a) of the rotation and scale of images, b) of the shift of images

duced, rotated and enlarged version of the images. Figure 5 shows the holographic image and its Fourier's spectrum in the Cartesian and log-polar coordinates.

Table 1 shows a preview of the results after registration. Successfully registered were not only shifted images (line 1), but also those rotated (line 2) and with the changed scale (line 3). Visual comparison of registered images (see Table 1) shows the correct positioning of the test image.

4 EVALUATION OF THE PROPOSED METHOD

The proposed procedure is suitable for registering holographic images, which were recorded during a laboratory experiment. Since these images are scanned in certain

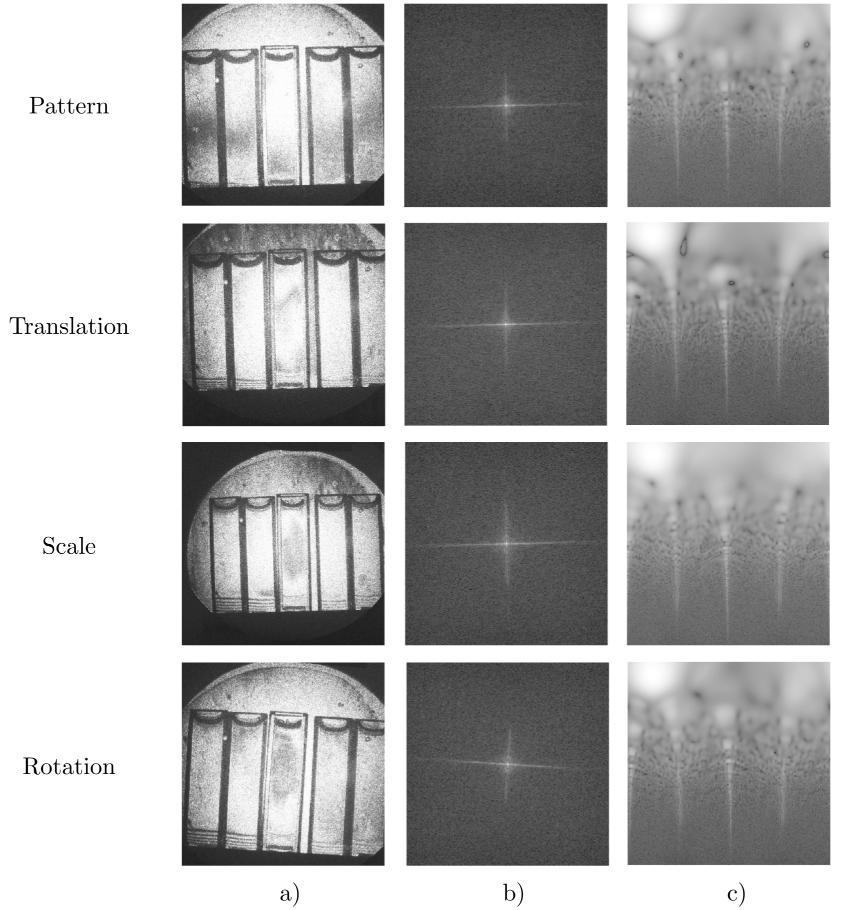


Fig. 5. Holographic image and its Fourier amplitude spectrum: a) input image, b) spectrum in Cartesian coordinates, c) spectrum in the log-polar coordinates

time intervals, there is a large amount of them and for subsequent evaluation it is necessary to have all these images equally geometrically registered, so we can apply more quantitative analysis to them.

Proposal of the method mentioned above resulted from a detailed study of available literature; the method of Fourier-Mellin transformation of images or of image registration with connection to holographic interferometry is not described in any of the available literature.

The proposed method shows the possibility of using image transformations, with the use of Fourier-Mellin transformation, when evaluating the experiments of holographic interferometry, resulting from the amplitude and frequency uniqueness of holograms. The use of discrete image transformations is original in this field and

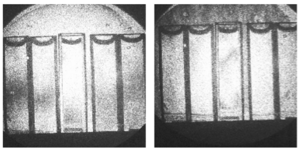
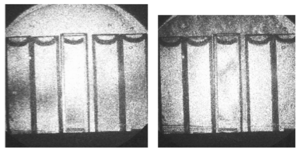
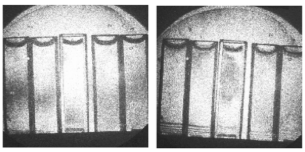
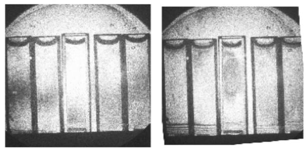
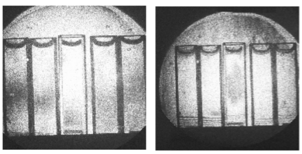
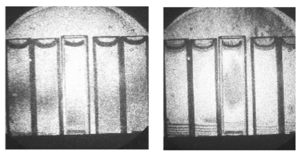
The reference and test image		Calculated transformation parameters
Before registration	After registration	
		translationX = 19 translationY = −41 scale = 1 rotation = 0
		translationX = 13 translationY = 18 scale = 1 rotation = 4.9
		translationX = 38 translationY = 0 scale = 0.83 rotation = 0

Table 1. Previews of registration

builds on previous original contribution of the author in the field of registration of fingerprints for authentication and identification of individuals using biometric identifiers. In addition to the normal sequence of applications of Fourier-Mellin transformation published in multiple contributions the proposed algorithm contains the original way of calculating the test image rotation angle; thus overall simplification and acceleration of the evaluation process and the registration of images is made.

4.1 The Disadvantages of the Method Used

Time complexity of computing: Registration of images with a shift, scaling, and rotation results in growing space of parameters of transformations. The proposed method is demanding in terms of computing time, because in the general case, when the images are rotated, shifted and have different scale, it is necessary for the image registration to calculate the following [15]: 6× fast Fourier transform (FFT), 3× inverse FFT, 2× log-polar resampling, 3× phase correlation, 2× filter, 2× rotation of the image.

Limited scale: Since large scale coefficients can change frequencies significantly, limited scale coefficients may be used [26].

The dimensions of the images: We can consider the need for adjustments to the size of the processed images under the disadvantage of the method for dimension 2^N, since fast Fourier transformation is used in numeric processing. Such an adjustment requires pre-processing of holographic images.

5 CONCLUSION

Registration is defined as finding the most appropriate geometric transformation, which describes the reciprocal space relationship between the reference and the input image. Images can be aligned through global coordinate transformations if the vector of parameters is found. Therefore the most important procedure in the registration of images is the determination of the transformation parameters from the images to be aligned.

The proposed registration of images allows registering the set of holographic interferograms to be prepared for further processing.

The application was created in Matlab environment and it is specialized for processing of holographic images obtained by the CCD camera during the measurements carried out in the laboratory using holographic interferometry.

The method based on phase correlation was used for registration of images. This method uses the fact that in case of displacement, rotation, and scale the integral transformations have their transforms in the frequency domain. Thus, on the basis of image transformation calculation it is possible to effectively determine the optimum registration.

After minor adjustments the proposed method can be used in the manufacturing process for automatic detection of errors in automation of the control of occurrence of defective products. In this case, the software should evaluate captured images of the products with the aim of determining the quantitative indicators of quality and reliability of the manufactured products.

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Pavol Božek has worked since 1978 as an independent researcher in the automotive industry. Since 1986 he is a lecturer at the Institute of Applied Informatics, Automation and Mathematics of the Faculty of Materials Science and Technology in Trnava, Slovak University of Technology Bratislava. His expertise focuses on automation of technological processes in industry, systems management and robotics. He is a member of the Scientific Committee of several conferences and Editorial Board of several journals. He took part in 8 scientific and educational projects, published 221 posts, including 10 monographs.

Elena Pivarčiová is working at the Department of Information Science and Automation Technology of Technical University in Zvolen as a University teacher in computer science, information technologies and programming since 2002. Currently, beside programming and holography she also works in biometrics and fingerprint verification using the Fourier-Mellin transformation. She took part in 10 scientific and educational projects, published 3 scientific monographs and 95 papers in national and international publications.